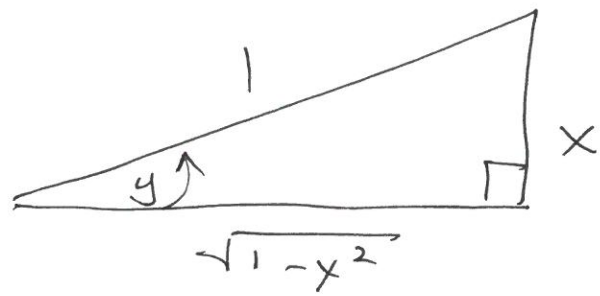
MA141-005

Monday, February 18

- continue... deriv. of inverse trig
- exponentials and logs – DERIV.
- log differentiation
- generalized power rule (proof)
- related rates (begin....)

(TEST #2: MONDAY, FEB. 25th (CH. 2))

INV. OF SINE:



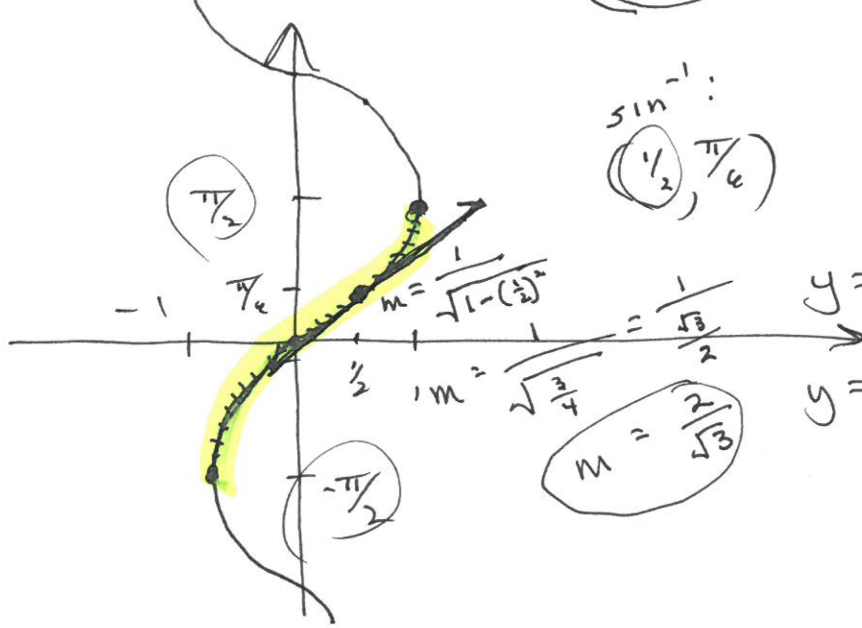
$x = \sin y$

$y = \sin^{-1} x$

$$y' = \frac{1}{\cos y} = \frac{1}{\frac{\sqrt{1-x^2}}{1}} = \frac{1}{\sqrt{1-x^2}} = y' = \underline{m_{TAN}}$$

$\sin x$: $(0,0)$, $(\frac{\pi}{2}, 1)$

$\sin^{-1}$: $(\frac{1}{2}, \frac{\pi}{6})$



$$m = \frac{1}{\sqrt{1-(\frac{1}{2})^2}} = \frac{1}{\sqrt{\frac{3}{4}}} = \frac{1}{\frac{\sqrt{3}}{2}}$$

$$m = \frac{2}{\sqrt{3}}$$

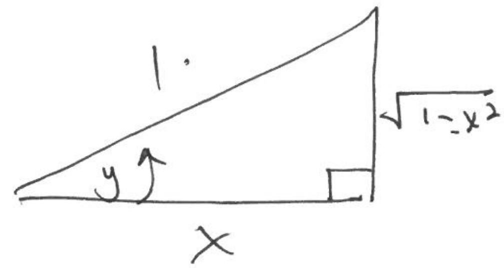
$y = \text{Sin}^{-1} x$

$y = \text{Arcsin}\, x$

$y = \cos x \qquad y' = -\sin x$

INV:

$x = \cos y$



$y = \cos^{-1} x \qquad y' = ? = \frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$

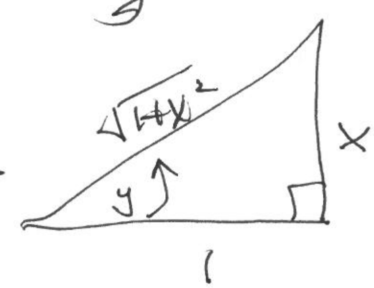
$x = \cos(y)$

$1 = -\sin y \cdot \frac{dy}{dx}$ (solve for $\frac{dy}{dx}$)

$\frac{-1}{\sin y} = \frac{dy}{dx} = \frac{-1}{\frac{\sqrt{1-x^2}}{1}} = \frac{-1}{\sqrt{1-x^2}}$

$y = \tan x \qquad y' = \sec^2 x$

INV:



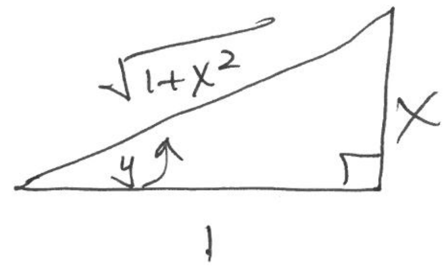
$x = \tan y$

$y = \tan^{-1} x \qquad y' = ??$

$x = \tan(y)$

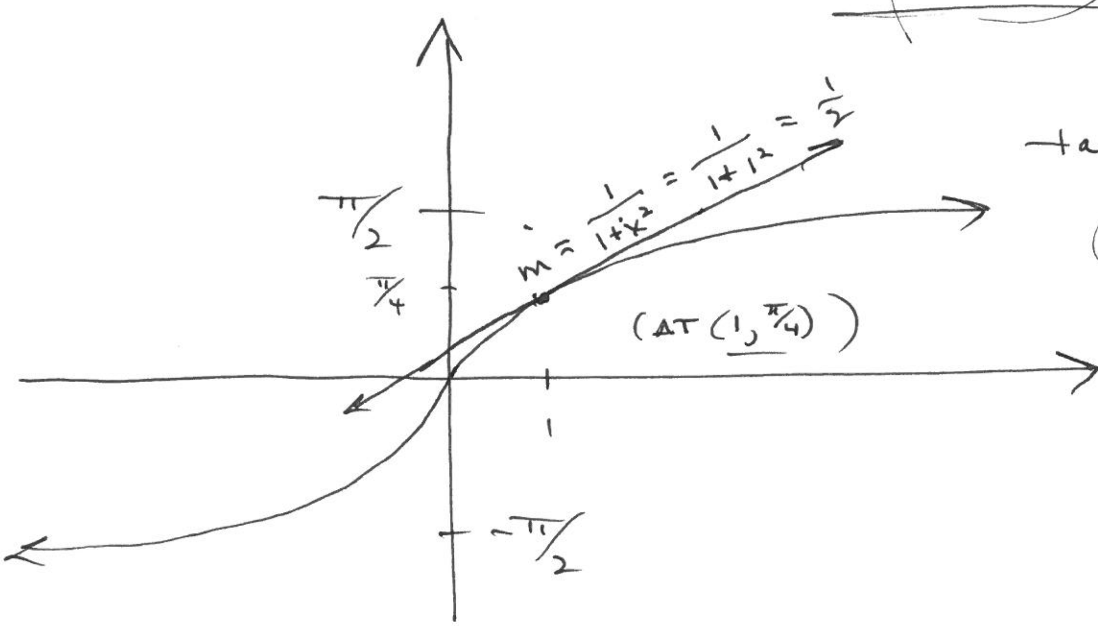
$1 = \sec^2 y \cdot \frac{dy}{dx}$ (solve for $\frac{dy}{dx}$)

$$\frac{1}{\sec^2 y} = \frac{dy}{dx}$$



$$\frac{1}{\left(\frac{\sqrt{1+x^2}}{1}\right)^2} = \frac{dy}{dx} = \frac{1}{1+x^2}$$

$$y = \tan^{-1} x \qquad y' = \frac{1}{1+x^2}$$



$\tan^{-1}(1) = \pi/4$

$(\tan \pi/4 = 1)$

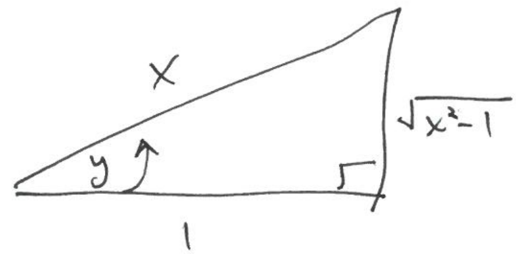
$y = \sec x \qquad y' = \sec x \cdot \tan x$

INV:

$x = \sec y$

$y = \sec^{-1} x$

$y' = ?$



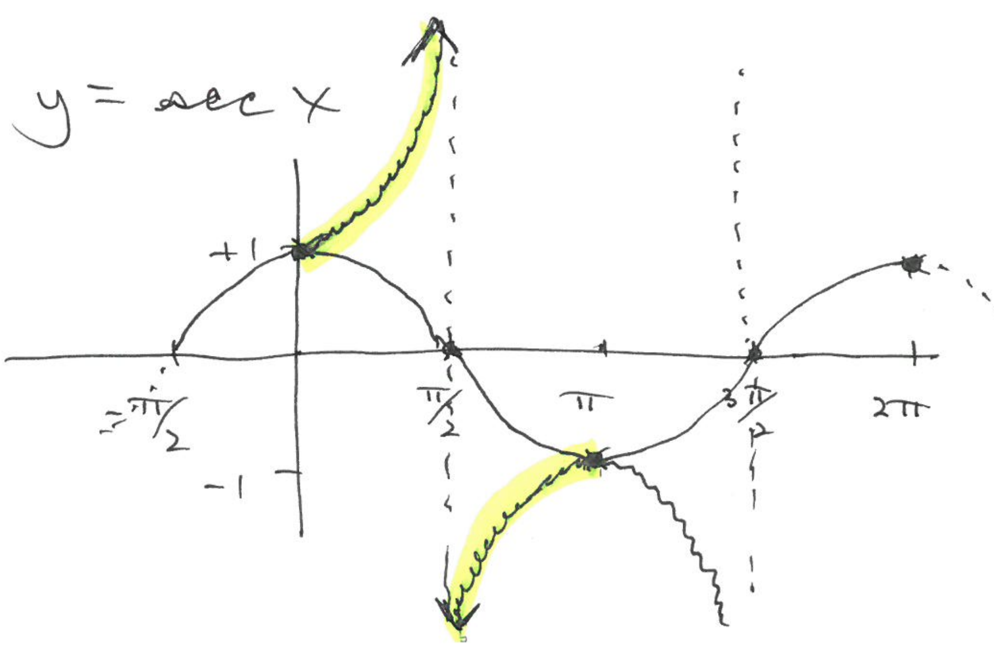
$x = \sec(y)$

$1 = \sec y \cdot \tan y \cdot \frac{dy}{dx}$ (solve for $\frac{dy}{dx}$

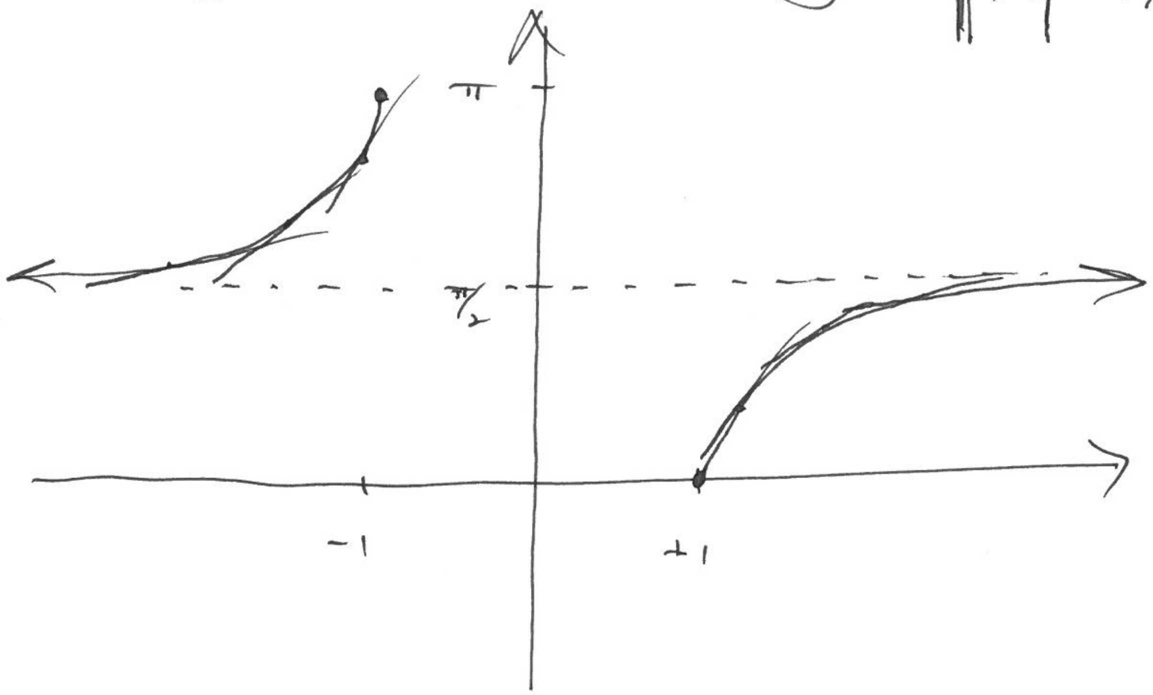
$\frac{1}{\sec y \cdot \tan y} = \frac{dy}{dx} = \frac{1}{x \cdot \frac{\sqrt{x^2-1}}{1}}$

$$\boxed{\frac{dy}{dx} = \frac{1}{x \cdot \sqrt{x^2-1}}}$$

$y = \sec x$



$y = \sec^{-1} x \qquad y' = \dfrac{1}{|x| \cdot \sqrt{x^2 - 1}}$



~~ba~~ base "e" $\boxed{e \approx 2.718}$

$$\lim_{k \to \infty} \underbrace{\left(1 + \frac{1}{k}\right)}_{1^\infty}{}^{k} = e$$

$k = 100$

$\left(1 + \frac{1}{100}\right)^{100} \approx 2.705$

$k = 10{,}000$

$\left(1 + \frac{1}{10{,}000}\right)^{10{,}000} \approx 2.718$

$k = 1{,}000{,}000$

$\left(1 + \frac{1}{1{,}000{,}000}\right)^{1{,}000{,}000} \approx 2.718$

$y = e^x$ (natural exponential function)

$y' = ??$

$$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$e^{x+h} = e^x \cdot e^h$

$$= \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$= \lim_{h \to 0} \frac{e^x [e^h - 1]}{h}$$

$$= e^x \cdot \lim_{h \to 0} \left[\frac{e^h - 1}{h}\right] \to 1$$

$$= e^x \cdot 1$$

$$y' = e^x$$

$h = .1$

$$\frac{e^{.1} - 1}{.1} \approx 1.05$$

$h = .01$

$$\frac{e^{.01} - 1}{.01} \approx 1.005$$

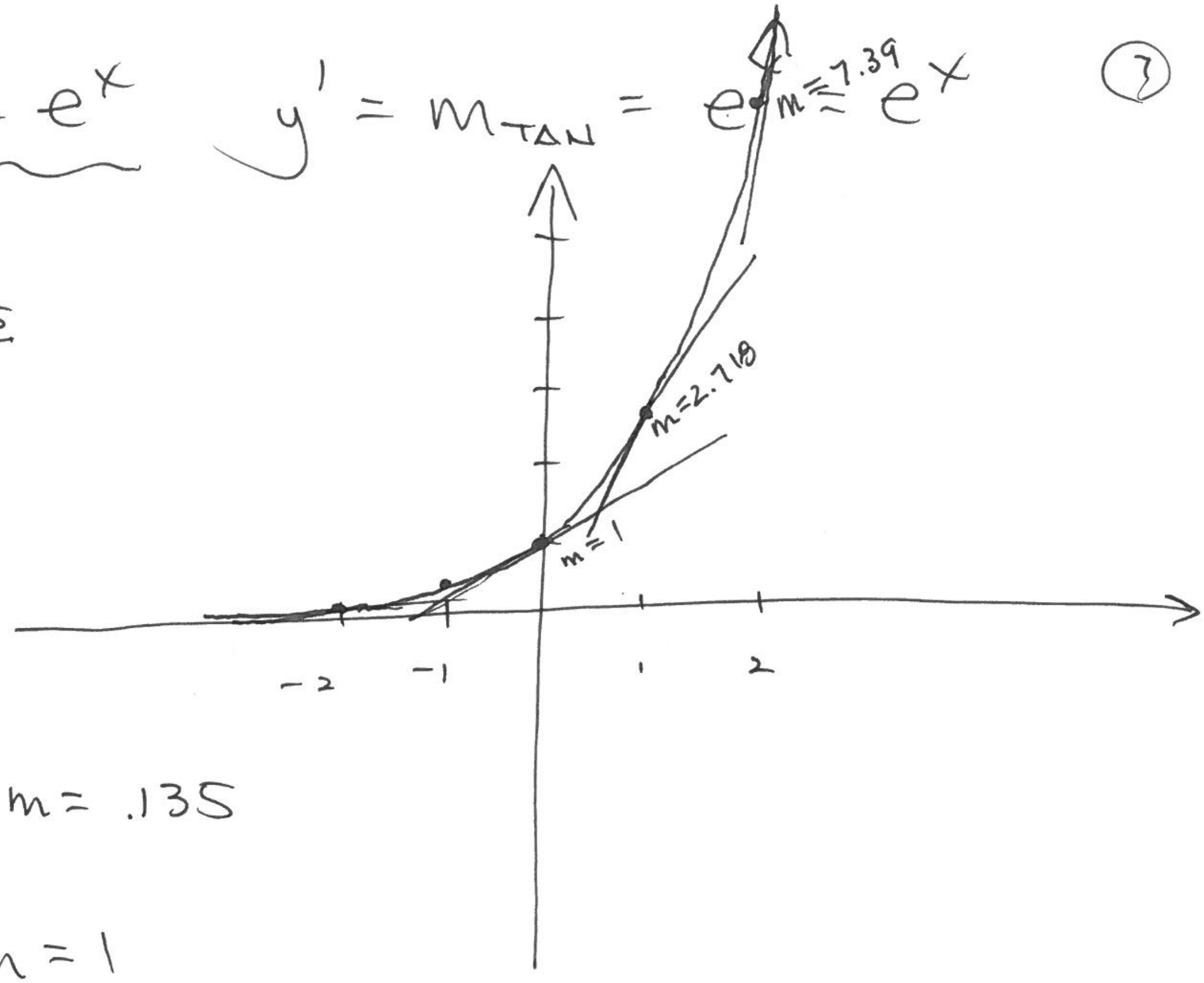
$h = .0001$

$$\frac{e^{.0001} - 1}{.0001} \approx 1.0005$$

$\vdots \qquad \vdots$

$h \to 0 \qquad 1$

$y = e^x \qquad y' = m_{TAN} = e^x$



x	y
-2	$e^{-2} \approx .135$
-1	$e^{-1} \approx .368$
0	$e^{0} = 1$
1	$e^{1} \approx 2.718$
2	$e^{2} \approx 7.39$

$(-2, .135)$ $m = .135$

$(-1, .368)$

$(0, 1)$ $m = 1$

$(1, 2.718)$

$(2, 7.39)$

$y = e^x \qquad y' = e^x$

$y = e^{(3x)} \qquad y' = e^{3x} \cdot 3 \leftarrow \frac{d(3x)}{dx}$

$y' = 3 \cdot e^{3x}$

$y = e^{8x^2+11x} \qquad y' = e^{8x^2+11x} \cdot (16x+11)$

$y' = (16x+11) \cdot e^{8x^2+11x}$

INV: $y = e^x \qquad y' = e^x$

$x = e^y$

y is the power to which "e" is raised to get x

$$y = \log_e x \quad (x = e^y)$$

$$y = \ln x \quad (\text{natural log})$$

$$y' = ?? = \frac{dy}{dx} = \frac{1}{x}$$

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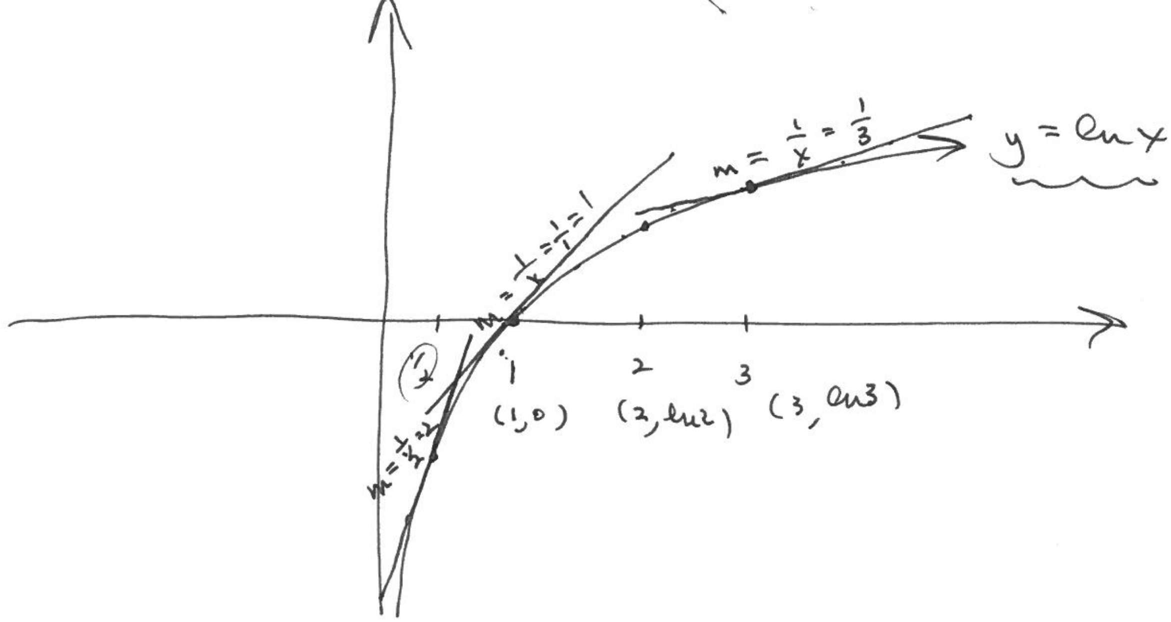
find y' (or $\frac{dy}{dx}$)

$$\boxed{\begin{array}{ll} y = e^x & y' = e^x \\ y = \ln x & y' = \frac{1}{x} \end{array}}$$

$$\underbrace{x} = e^y$$

$$1 = e^y \cdot \frac{dy}{dx} \quad (\text{solve for } \frac{dy}{dx})$$

$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}.$$



$y = \ln(x) \qquad y' = \frac{1}{x}$

$y = \ln(3x^2 - 4x) \qquad y' = \frac{1}{3x^2-4x}(6x-4)$

$$y' = \frac{6x-4}{3x^2-4x}$$

$y = 2^x \qquad y = 5^x \qquad y = 10^x$

general exponential

$\boxed{y = 5^x}$ rewrite in terms of base "e"

$\ln_e 5 = 1.609$

$(e^{1.609} = 5)$

$e^{\ln 5} = e^{\text{the power you would raise e to get 5}}$

$= 5$

$e^{\ln 34} = 34$

$\boxed{e^{\ln u} = u}$ ✱ ✱

$y = 5^x = e^{\ln 5^x} = e^{x \cdot \ln 5}$

$$y = 5^x = e^{\ln 5^x} = e^{x \cdot \ln 5}$$

$$y' = ??$$

$d(x \cdot 3) = 3$

$d(x \cdot 8) = 8$

$$y = e^{x \cdot \ln 5}$$

$$y' = e^{x \cdot \ln 5} \cdot \frac{d(x \cdot \ln 5)}{dx}$$

$$y' = e^{x \cdot \ln 5} (\ln 5)$$

$$\boxed{y' = 5^x \cdot (\ln 5)}$$

$$\boxed{y = a^x \qquad y' = a^x \cdot \ln a} \qquad a > 1$$

$$y = e^x \qquad y' = e^x \cdot \ln e = e^x$$

$$y = 8^x \qquad y' = 8^x \cdot \ln 8$$

chain rule

$$y = 10^{x^2+x} \qquad y' = 10^{x^2+x} \cdot \ln 10 \cdot (2x+1)$$

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generalized power rule:

$$y = x^p \qquad y' = p \cdot x^{p-1} \text{ (for any } p)$$

$$y = e^{\ln x^p} = e^{p \cdot \ln x}$$

$$y' = e^{p \cdot \ln x} \cdot p \cdot \frac{1}{x} = x^p \cdot p \cdot \left(\frac{1}{x}\right) = \boxed{x^{p-1} \cdot p}$$

log diff.

$$y = \frac{[x^2+x+3]^7 \cdot \sqrt[5]{8x+1}}{(42x+3)^3}$$

$\ln(a)^r = r \cdot \ln a$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$\ln(a \cdot b) = \ln a + \ln b$

take LN of both sides . . . .

then DIFFERENTIATE

$$\ln y = \ln\left[\frac{(x^2+x+3)^7 \cdot (8x+1)^{1/5}}{(42x+3)^3}\right]$$

$$\ln y = \ln(x^2+x+3)^7 + \ln(8x+1)^{1/5} - \ln(42x+3)^3$$

$$\ln y = 7 \cdot \ln(x^2+x+3) + \frac{1}{5}\ln(8x+1) - 3\ln(42x+3)$$

(take DERIV.)

$$\frac{1}{y} \cdot \frac{dy}{dx} = 7 \cdot \frac{1}{(x^2+x+3)}(2x+1) + \frac{1}{5} \cdot \frac{1}{(8x+1)}(8) - 3 \cdot \frac{1}{(42x+3)}(42)$$