MA141-005

Wednesday, February 13

- today finish chain rule
- implicit differentiation
- deriv of inverse trig.

Chain Rule

(2) composition of functions:

$$y = (f \circ g)(x) = f(g(x))$$

$$y' = ??$$

$$y' = f'(g(x)) \cdot g'(x)$$

ex: $f(x) = x^5$

$g(x) = 2x^2 - x + 7$

$$y = f(g(x)) = f(2x^2-x+7) = (2x^2-x+7)^5$$

$$y' = f'(g(x)) \cdot g'(x)$$

$$y' = 5(2x^2-x+7)^4 \cdot (4x-1)$$

DERIV:

$$\boxed{y' = 5(2x^2-x+7)^4 \cdot (4x-1)}$$

(2)

(3) y in terms of u

u in terms of x

$y = 6u^4$

$u = 8x - 3$

$\frac{dy}{du} = 24u^3$

$\frac{du}{dx} = 8$

$$\boxed{\frac{dy}{du} \cdot \frac{du}{dx}} = \frac{dy}{dx}$$

$$(24u^3) \cdot (8) = \frac{dy}{dx}$$

$$\boxed{(24(8x-3)^3) \cdot (8) = \frac{dy}{dx}}$$

check:

$$y = 6u^4 \quad (u = 8x-3)$$

$$y = 6\boxed{(8x-3)^4}$$

$$\boxed{y' = 6\left[4(8x-3)^3 \cdot 8\right]}$$

$\frac{dy}{dx}$

chain:

$$\frac{dy}{dr} \cdot \frac{dr}{dw} \cdot \frac{dw}{dq} \cdot \frac{dq}{dx} =$$

$$f(\theta) = \sqrt{\tan\theta + \cot\theta} = (\tan\theta + \cot\theta)^{1/2}$$

$$f'(\theta) = \frac{1}{2}(\tan\theta + \cot\theta)^{-1/2} \cdot \left[\sec^2\theta + (-\csc^2\theta)\right]$$

$$f'(\theta) = \frac{\sec^2\theta - \csc^2\theta}{2\sqrt{\tan\theta + \cot\theta}}$$

$$f'(\pi/4) =$$

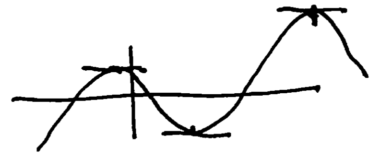
$$\cos\theta \cdot \sec\theta$$

$$f(x) = \frac{2 \cdot \boxed{\sin(5x)}}{1 - 3 \cdot \boxed{\sec(5x)}}$$

$$f'(x) = \frac{(1-3\sec 5x)\cdot(2\cdot\cos 5x\cdot 5) - (2\sin 5x)(0 - 3\sec 5x\tan 5x \cdot 5)}{(1-3\sec 5x)^2}$$

$$f'(x) = \frac{(1-3\sec 5x)(10\cos 5x) + 30\sin 5x\sec 5x\tan 5x}{(1-3\sec 5x)^2}$$

(4)

find the points on 

$y = (4x^2 - 8x + 3)^4$ at which

the tangent line is horizontal

$$m_{TAN} = \boxed{0 = y'}$$

$$y' = \boxed{4\left(4x^2 - 8x + 3\right)^3 \cdot (8x - 8) = 0}$$

$4x^2 - 8x + 3 = 0$

$\cancel{(4x \quad)(x \quad) = 0}$

$(2x - 3)(2x - 1) = 0$

$2x - 3 = 0 \qquad 2x - 1 = 0$

$2x = 3 \qquad 2x = 1$

$x = 3/2 \qquad x = 1/2$

$8x - 8 = 0$

$8x = 8$

$x = 1$

$(1, f(1)) = (1, 1)$

$f(1) = (4 \cdot 1^2 - 8 \cdot 1 + 3)^4$

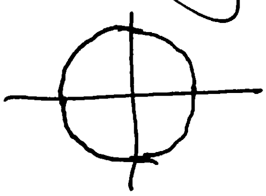
$f(1) = (-1)^4 = 1$

$(3/2, f(3/2))$ & $(\frac{1}{2}, f(1/2))$

$(3/2, 0) \qquad (1/2, 0)$

implicit differentiation

$x^2 + y^2 = 1$

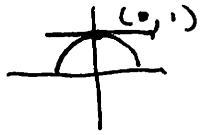


circle; center $(0,0)$ $r=1$

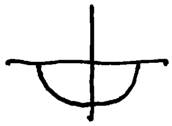
solve for y

$y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2}$



① $y = \sqrt{1-x^2}$ upper $\frac{1}{2}$-circle



② $y = -\sqrt{1-x^2}$ lower $\frac{1}{2}$-circle

$m = \frac{-x}{y}$

① $y = (1-x^2)^{1/2}$ $\quad y' = \frac{1}{2}(1-x^2)^{-1/2}(-2x) = \boxed{\frac{-x}{\sqrt{1-x^2}} = y'}$

m_{TAN} @ $(0,1)$: $m = \frac{-0}{\sqrt{1-0^2}} = 0$

② $y = -(1-x^2)^{1/2}$ $\quad y' = -1 \cdot \frac{1}{2}(1-x^2)^{-1/2}(-2x) = \boxed{\frac{x}{-\sqrt{1-x^2}} = y'}$

$x^2 + y^2 = 1$

$\boxed{\text{find } \frac{dy}{dx}}$

$(x)^2 + (y)^2 = 1$

$2(x)^1 \frac{dx}{dx} + 2(y)^1 \cdot \frac{dy}{dx} = 0$

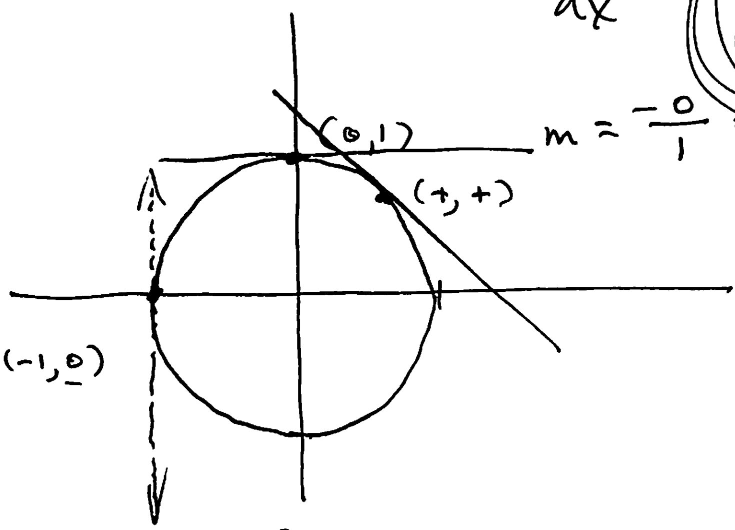
$\boxed{2x + 2y \cdot \frac{dy}{dx} = 0}$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$\frac{\cancel{2y} \cdot \frac{dy}{dx}}{\cancel{2y}} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$m = \frac{-0}{1} = 0$



$m = \frac{-(-1)}{0} \rightarrow$ undef.

$$3x \cdot y^4 - \frac{x}{y} = 4$$

$$\boxed{\text{find } \frac{dy}{dx}:}$$

$$\left[(3x)\cdot\frac{d(y)^4}{dx} + (y^4)\cdot\frac{d(3x)}{dx}\right] - \frac{(y)\frac{d(x)}{dx} - (x)\cdot\frac{d(y)}{dx}}{y^2} = 0$$

$$y^2\left[(3x)\left[4\cdot y^3\cdot\frac{dy}{dx}\right] + (y)^4\cdot 3\right] - \left(\frac{y\cdot(1) - x\cdot\frac{dy}{dx}}{\cancel{y^2}}\right)\cancel{y^2} = 0\cdot y^2$$

$$4\cdot 3x\,y^5\cdot\frac{dy}{dx} + 3y^6 - \left(y - x\cdot\frac{dy}{dx}\right) = 0$$

$$12xy^5\cdot\frac{dy}{dx} + 3y^6 - y + x\cdot\frac{dy}{dx} = 0$$

$$12xy^5\cdot\frac{dy}{dx} + x\cdot\frac{dy}{dx} = y - 3y^6$$

$$\frac{\frac{dy}{dx}\cancel{\left[12xy^5 + x\right]}}{\cancel{\left[12xy^5 + x\right]}} = \frac{y - 3y^6}{\left[12xy^5 + x\right]}$$

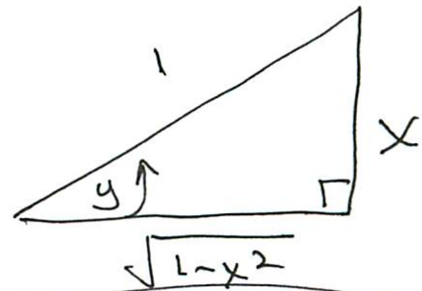
$$\frac{dy}{dx} = \frac{y - 3y^6}{12xy^5 + x}$$

DERIVATIVES (INVERSE TRIG)

$y = \sin x$ $\qquad$ $y' = \cos x$

INV:

$\boxed{x = \sin y}$ →



$y = \arcsin x$

$y = \sin^{-1} x$ $\qquad$ $y' = ?? = \frac{1}{\sqrt{1-x^2}}$

$x = \sin y$ $\qquad$ find $\frac{dy}{dx}$:

$1 = \cos y \cdot \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\frac{\sqrt{1-x^2}}{1}} = \frac{1}{\sqrt{1-x^2}}$$