Monday, February 11

- DERIV OF SIN, COS (my def.)
- DERIV OF TAN, COT, SEC, CSC
- BEGIN CHAIN RULE

$$\boxed{\cos\theta < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}}$$

as $\theta \to 0$

$$1 < \frac{\theta}{\sin\theta} < 1$$

$$\lim_{\theta \to 0} \frac{\theta}{\sin\theta} = 1$$

$$\frac{1}{\cos\theta} > \frac{\sin\theta}{\theta} > \cos\theta$$

$\theta \to 0$

$$1 > \frac{\sin\theta}{\theta} > 1$$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

$$\boxed{\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0}$$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$$\lim_{\theta \to 0} \frac{\boxed{(\cos\theta - 1)(\cos\theta + 1)}}{\theta\,(\cos\theta + 1)} =$$

$$\lim_{\theta \to 0} \frac{\cos^2\theta - 1}{\theta(\cos\theta + 1)} = \lim_{\theta \to 0} \frac{-\sin^2\theta}{\theta(\cos\theta + 1)}$$

$$= \lim_{\theta \to 0} \frac{-\sin\theta}{(\cos\theta + 1)} \cdot \boxed{\frac{\sin\theta}{\theta}}^{\to 1} = \frac{-0}{2} \cdot 1$$

$$= 0$$

DEF OF DERIV....

DERIV OF SIN X

$f(x) = \sin x$

$$f'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

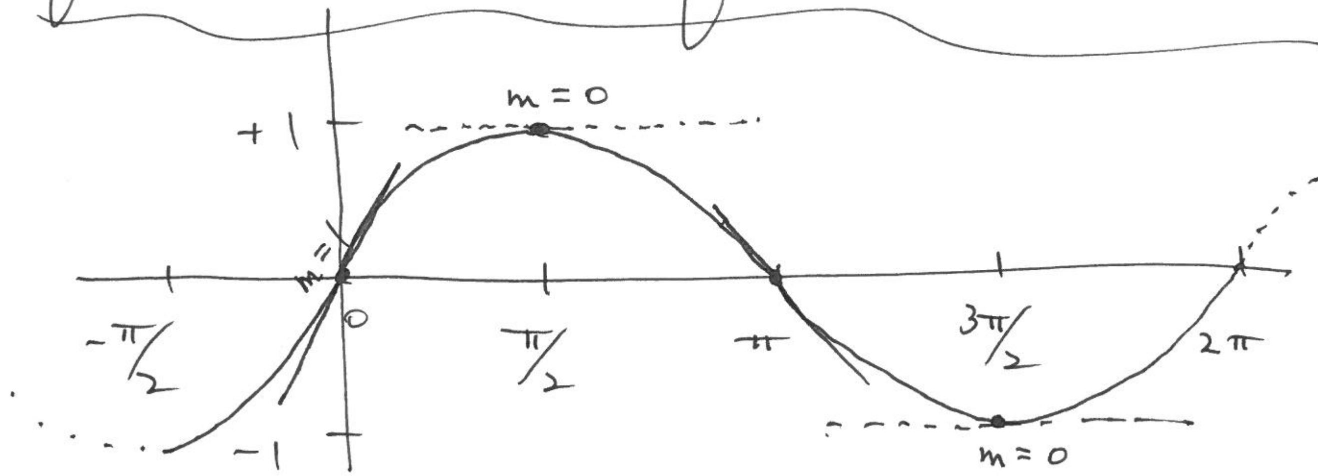
$$= \lim_{h \to 0} \frac{[\sin x \cosh + \cos x \cdot \sinh] - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{[\sin x \cdot \cos h - \sin x]}{h} + \frac{\cos x \cdot \sin h}{h}$$

$$= \lim_{h \to 0} \sin x \frac{[\cos h - 1]}{h}^{\to 0} + \cos x \cdot \frac{\sin h}{h}^{\to 1}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \boxed{\cos x = f'(x)}$$

$f(x) = \sin x \qquad f'(x) = \cos x = m_{TAN}$



① at $x = \pi/2$

$m_{TAN} = f'(\pi/2) = \cos \pi/2 = 0$

② at $x = 3\pi/2$

$m_{TAN} = f'(3\pi/2) = \cos 3\pi/2 = 0$

③ at $x = 0$

$m_{TAN} = f'(0) = \cos 0 = 1$

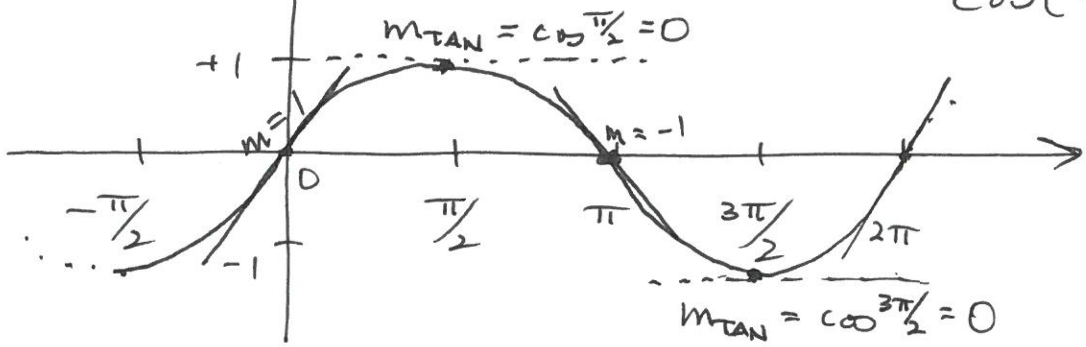
DEF OF DERIV:

$$\boxed{f(x) = \sin x}$$

$$\boxed{f'(x) = \cos x} = m_{TAN}$$

① at $\pi/2$

$$\cos(\pi/2) = m_{TAN}$$



② at 0

$$m_{TAN} = \cos(0) = 1$$

$$f(x) = \cos x \qquad f'(x) = ??$$

$$\boxed{\cos(A+B) = \cos A \cos B - \sin A \sin B}$$

$$f'(x) = \lim_{h \to 0} \frac{\boxed{\cos(x+h)} - \cos x}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cos x \cdot \cos h - \sin x \sin h - \cos x}{h}$$

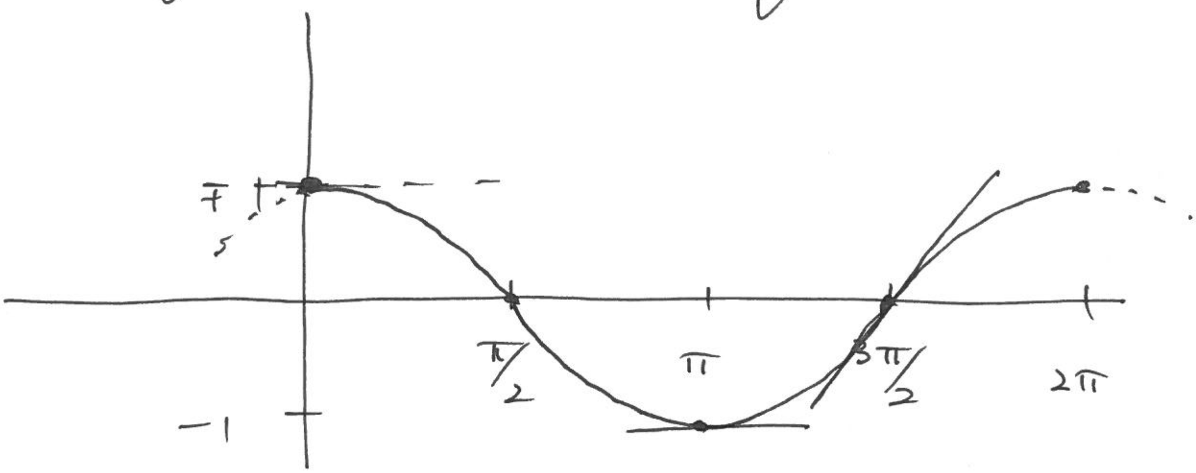
$$f'(x) = \lim_{h \to 0} \left[\frac{\cos x \cdot \cos h - \cos x}{h} - \frac{\sin x \sin h}{h} \right]$$

$$= \lim_{h \to 0} \left[\cos x \boxed{\frac{(\cos h - 1)}{h}} \to 0 - (\sin x) \boxed{\frac{(\sin h)}{h}} \to 1 \right]$$

$$= (\cos x)(0) - (\sin x)(1)$$

$$f'(x) = -\sin x$$

$f(x) = \cos x \qquad f'(x) = -\sin x$



① at $x = 0$

$f'(x) = m_{TAN} = -\sin 0 = 0$

② at $x = \pi$

$f'(x) = m_{TAN} = -\sin \pi = 0$

③ at $x = 3\pi/2$

$f'(x) = m_{TAN} = -\boxed{\sin 3\pi/2} = -(-1) = 1$

$$\frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

$y = \tan x \qquad y' = ??$

$y = \dfrac{\sin x}{\cos x} \qquad y' = ??$

quotient rule:

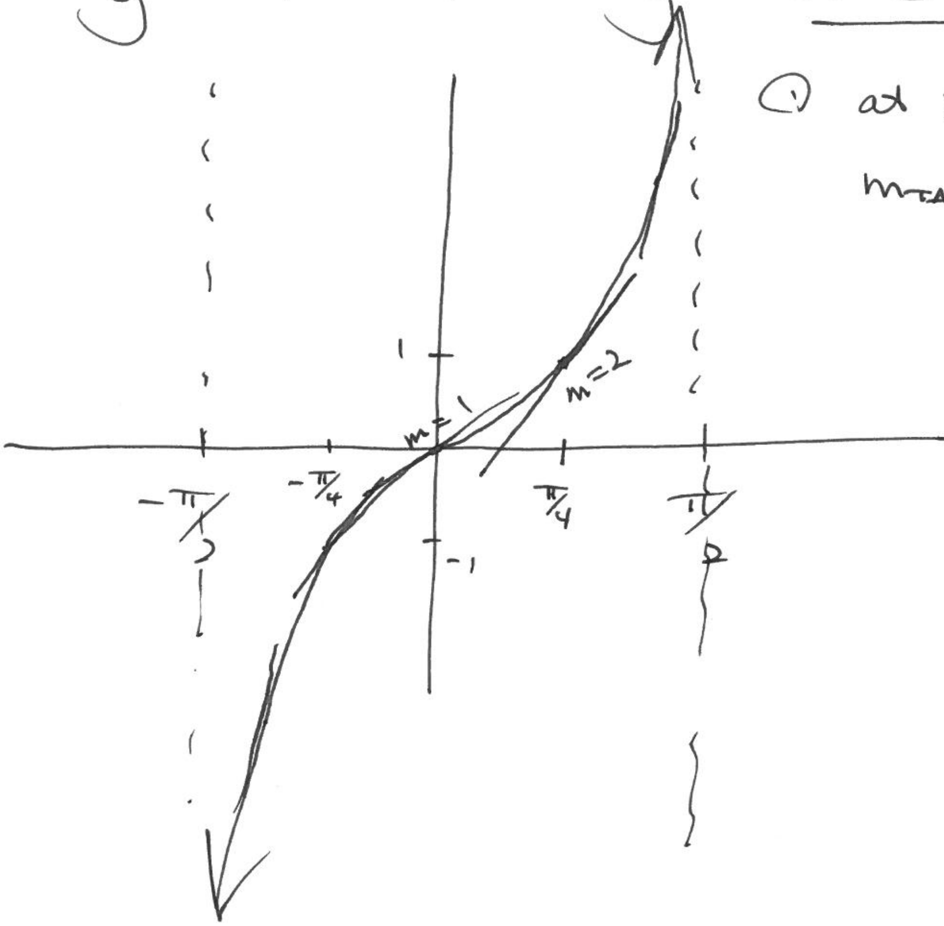
$$y' = \frac{(\cos x)\cdot d(\sin x) - (\sin x)\cdot d(\cos x)}{[\cos x]^2}$$

$$y' = \frac{(\cos x)\cdot(\cos x) - (\sin x)(-\sin x)}{[\cos x]^2}$$

$$y' = \frac{(\cos x)(\cos x) + (\sin x)(\sin x)}{[\cos x]^2}$$

$$y' = \frac{1}{(\cos x)^2} = (\sec x)^2 = \sec^2 x$$

$y = \tan x \qquad y' = \sec^2 x$



① at $x = \pi/4$

$m_{TAN} = f'(\pi/4) = (\sec \pi/4)^2$

$= \left(\frac{1}{\cos \pi/4}\right)^2 = \left(\frac{2}{\sqrt{2}}\right)^2$

$= \frac{4}{2} = 2$

$y = \cot x = \frac{\cos x}{\sin x} \qquad y' = ?? = -\csc^2 x$

$$y' = \frac{(\sin x) \cdot d(\cos x) - (\cos x) \cdot d(\sin x)}{[\sin x]^2}$$

$$y' = \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{[\sin x]^2}$$

$$y' = \frac{-\sin^2 x - \cos^2 x}{(\sin x)^2} = \frac{-1\,(\overbrace{\sin^2 x + \cos^2 x}^{1})}{(\sin x)^2}$$

$$y' = \frac{-1}{(\sin x)^2} = -\csc^2 x = -(\csc x)^2$$

$f(x) = \sec x = \dfrac{1}{\cos x}$ $\quad f'(x) = ??$

$$f'(x) = \frac{\cancel{(\cos x)\cdot(0)} - (1)(-\sin x)}{(\cos x)^2}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \sec x \cdot \tan x$$

$f(x) = \sec x \quad f'(x) = \sec x \cdot \tan x$

$f(x) = \csc x = \dfrac{1}{\sin x}$

$$f'(x) = \frac{\cancel{(\sin x)\cdot(0)} - (1)(\cos x)}{(\sin x)^2}$$

$$f'(x) = \frac{-\cos x}{\sin x} \cdot \frac{1}{\sin x} = -\csc x \cdot \cot x$$

from ex. 2.4:

$$\lim_{\beta \to 0} \frac{\beta}{\sin \beta} = 1$$

5θ

2.) $\lim_{\theta \to 0} \frac{\theta^2}{\sin 5\theta} =$

$$= \lim_{\substack{\theta \to 0 \\ \boxed{5\theta \to 0}}} \frac{\theta \cdot \theta \cdot 5}{5 \sin 5\theta} \to 1 = \frac{\theta \cdot 1}{5} = 0$$

10.) find $\frac{dy}{dx}$:

$$y = \boxed{3x \cdot \sin x} + \frac{\sqrt{x}}{\cos x} \to \frac{x^{1/2}}{\cos x}$$

prod. rule: quotient rule

$$y' = \frac{dy}{dx} = \left[(3x) \cdot (\cos x) + (\sin x) \cdot (3)\right] +$$

$$\frac{(\cos x) \cdot \left(\frac{1}{2} \cdot x^{-1/2}\right) - (x^{1/2}) \cdot (-\sin x)}{(\cos x)^2}$$

$$\frac{dy}{dx} = 3x \cos x + 3 \sin x + \left[\frac{(\cos x)}{2\sqrt{x}} + \frac{\sqrt{x} \cdot \sin x \cdot 2\sqrt{x}}{2\sqrt{x}}\right] \cdot \frac{1}{\cos^2 x}$$

$$\frac{dy}{dx} = 3x\cos x + 3\sin x + \frac{\cos x + 2x \sin x}{2\sqrt{x} \cdot \cos^2 x}$$

20.) find the eq. of the tangent line $\left(\frac{\pi}{4}, \frac{1}{2}\right)$

$y = \underline{\sin x \cdot \cos x}$ (at $x = \frac{\pi}{4}$)

$y = \sin\frac{\pi}{4} \cdot \cos\frac{\pi}{4}$
$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{1}{2}$

$y - \underline{y_1} = m(x - x_1)$

$y - \frac{1}{2} = 0 \left(x - \frac{\pi}{4}\right)$

$y - \frac{1}{2} = 0 \qquad y = \frac{1}{2}$

$m = m_{TAN} = \text{DERIV} = 0$

$y' = (\sin x)(-\sin x) + (\cos x)(\cos x)$

$y' = -\sin^2 x + \cos^2 x$

$y' = -\left[\frac{\sqrt{2}}{2}\right]^2 + \left[\frac{\sqrt{2}}{2}\right]^2 = 0$

2.5: CHAIN RULE

$y = x^n \qquad y' = n \cdot x^{n-1}$

(power rule)

$y = [f(x)]^n$ ex: $y = (3x^2 - x + 4)^5$

ex: $y = (2x+1)^2$

power rule:

$y' \overset{?}{=} 2(2x+1)^1 = 4x+2$

$y = (2x+1)^2 = \underline{4x^2} + \underline{4x} + \underline{1}$

$y' = 8x+4$

?

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① EXTENDED POWER RULE:

$y = [f(x)]^n \Rightarrow y' = n[f(x)]^{n-1} \cdot f'(x)$

deriv of inside function

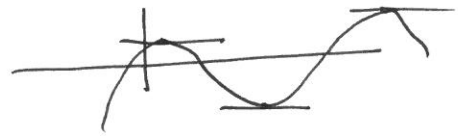
ex: $y = (2x+1)^2$

$y' = 2(2x+1)^1 \cdot (2) = 8x+4$

ex: $y = (3x^2 - x + 4)^5$

$$\begin{cases} y' = 5(3x^2 - x + 4)^4 \cdot (6x - 1) \\ y' = (30x - 5) \cdot (3x^2 - x + 4)^2 \end{cases}$$

ex:



$$y = \left[\frac{2x+1}{3x-8}\right]^3$$

$$y' = 3\left[\frac{2x+1}{3x-8}\right]^2 \cdot \left[\frac{\overset{\cancel{6x}-16\;-\cancel{6x}-3}{(3x-8)(2)-(2x+1)(3)}}{(3x-8)^2}\right] *$$

$$y' = \frac{3}{1} \cdot \frac{(2x+1)^2}{(3x-8)^2} \cdot \frac{-19}{(3x-8)^2}$$

$$\boxed{y' = \frac{3(-19)(2x+1)^2}{(3x-8)^4}}$$

① $y' = 0$

② y' undef.

MA141-004

TEST #1 RESULTS

A's:	7	(18%)	33%
B's:	6	(15%)	
C's:	9	(23%)	
D's:	4	(10%)	43%
F's:	13	(33%)	

AVE: 66.63%