MA 141 - 005

Wednesday, February 6

today:

- quick review from videos (deriv of constant; deriv of linear; deriv of $k \cdot f(x)$; deriv of sum/diff; power rule; generalized power rule)
- product rule
- quotient rule
- higher derivatives
- "normal" line
- deriv of TRIG (intro...)

PRODUCT RULE:

$$y = f(x) \cdot g(x) \qquad y' = ??$$

$$y' = \lim_{h \to 0} \frac{f(x+h) \cdot g(x+h) - f(x) \cdot g(x)}{h}$$

$$y' = \lim_{h \to 0} \left[\frac{f(x+h) \cdot g(x+h) - f(x) \cdot g(x+h) + f(x) \cdot g(x+h) - f(x) \cdot g(x)}{h}\right]$$

$$y' = \lim_{h \to 0} \boxed{g(x+h)} \frac{[f(x+h) - f(x)]}{h} + f(x) \frac{[g(x+h) - g(x)]}{h}$$

$$\boxed{y' = g(x) \cdot f'(x) + f(x) \cdot g'(x)} \quad ** $$

or $\boxed{y' = f(x) \cdot g'(x) + g(x) \cdot f'(x)}$ ✓

ex: $\boxed{y = (2x-1)(5x+4)}$

$$y' = m_{TAN} = (2x-1)(5) + (5x+4)(2)$$

$$y' = 10x - 5 + 10x + 8$$

$$y' = 20x + 3$$

$$f'(2) = 20(2) + 3 = 43$$

~~QUO~~

$$y = \left(42x^2 - 11\sqrt{x} + \frac{8}{x^3}\right)\left(5x^{-2/3} - 41x^{1/2} - 11x\right)$$

$$y = 10x^2 - 5x + 8x - 4$$

$$y = 10x^2 + 3x - 4$$

$$y' = 20x + 3$$

QUOTIENT RULE:

$$y = \frac{f(x)}{g(x)} \qquad y' = ??$$

$$y' = \lim_{h \to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

$$y' = \lim_{h \to 0} \left[\frac{f(x+h) \cdot g(x)}{g(x+h) \cdot g(x)} - \frac{f(x)}{g(x)} \cdot \frac{g(x+h)}{g(x+h)}\right] \cdot \frac{1}{h}$$

$$y' = \lim_{h \to 0} \frac{f(x+h) \cdot g(x) - f(x) \cdot g(x+h)}{g(x+h) \cdot g(x) \cdot h}$$

$$y' = \lim_{h \to 0} \frac{\left[f(x+h) \cdot g(x) - f(x) \cdot g(x)\right] + \left[f(x)\, g(x) - f(x) \cdot g(x+h)\right]}{g(x+h) \cdot g(x) \cdot h}$$

$$y' = \lim_{h \to 0} \frac{g(x)\left[f(x+h) - f(x)\right] + f(x)\left[g(x) - g(x+h)\right]}{g(x+h) \cdot g(x) \cdot h}$$

$$y' = \lim_{h \to 0} \frac{g(x)\left[f(x+h) - f(x)\right]}{\boxed{g(x+h)} \cdot g(x) \cdot h} + \frac{(-f(x))\left[g(x+h) - g(x)\right]}{\boxed{g(x+h)} \cdot g(x) \cdot h}$$

$$y' = \cancel{\lim_{h \to 0}} \frac{g(x) \cdot f'(x)}{[g(x)]^2} - \frac{f(x) \cdot g'(x)}{[g(x)]^2}$$

$$\boxed{y' = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}}$$

QUOTIENT RULE

$$\boxed{y = \frac{f(x)}{g(x)}}$$

$$y = \frac{(2x-1)}{(5x+4)}$$

$$y' = \frac{(5x+4)\cdot(2) - (2x-1)\cdot(5)}{(5x+4)^2} = m_{TAN}$$

$$y' = \frac{10x+8-(10x-5)}{(5x+4)^2}$$

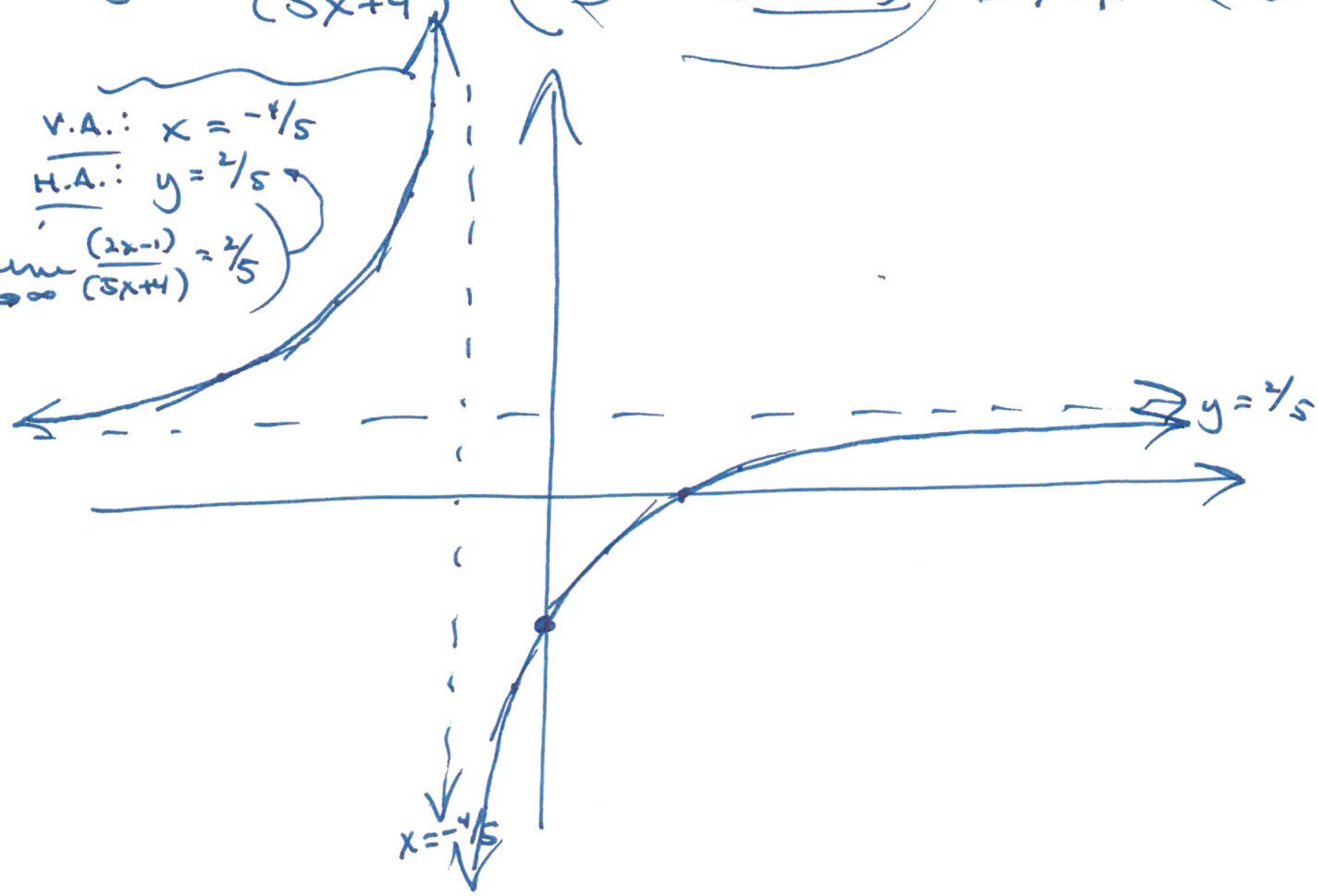
$$y' = \frac{\cancel{10x}+8 \cancel{-10x}+5}{(5x+4)^2} = \frac{13}{(5x+4)^2} = m_{TAN}$$

$$y = \frac{(2x-1)}{(5x+4)} \qquad y' = \frac{13}{(5x+4)^2} \begin{matrix} \rightarrow + \\ \rightarrow + \end{matrix} \; \frac{+}{+} = +$$

V.A.: $x = -4/5$

H.A.: $y = 2/5$

$\left(\lim_{x\to\infty} \frac{(2x-1)}{(5x+4)} = 2/5\right)$



HIGHER ORDER DERIVATIVES:

$y \cdots y' \cdots y'' \cdots y''' \cdots y^{(4)}$

$\uparrow$ 4th DERIV

$f(x) \cdots f'(x) \cdots f''(x) \cdots$

$\frac{dy}{dx} \cdots \frac{d^2y}{dx^2} \cdots \frac{d^3y}{dx^3} \cdots$

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ex:

$y = 8x^3 + 5x^2 - 11x + \underline{2}$

$y' = 8(3x^2) + 5(2x) - 11(1) + 0$

$y' = 24x^2 + 10x \underline{-11}$

$y'' = 24(2x) + 10(1) - 0$

$y'' = 48x + 10$

$y''' = 48$

$s(t) = -16t^2 + 64t + 100$ free falling object

$s(t)$: ft

t: sec

DIST; HT; POS

1. $t=0$ $s(0) = -16(0)^2 + 64(0) + 100 = 100$ ft.

$t=1$ $s(1) = -16(1)^2 + 64(1) + 100 = 148$ ft

$t=2$

2. $s'(t) = v(t) = -16(2t) + 64 + 0$

$\boxed{v(t) = -32t + 64}$

$t=0$ $v(0) = -32(0) + 64 = 64 \frac{ft}{sec}$ GRAV

$t=1$ $v(1) = -32(1) + 64 = 32$ ft/sec

$t=2$

3. $s''(t) = v'(t) = -32 \frac{ft/sec}{sec} = -32$ ft/sec^2

GRAV

$v'(t) = a(t) = -32$

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normal:

$y = x^3 - 3x$

$y' = 3x^2 - 3 = m_{TAN}$

$m_{NORMAL} = \frac{-1}{3x^2-3}$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

// to $\boxed{2x + 18y - 9 = 0}$

$\frac{18y}{18} = \frac{-2x}{18} - \frac{9}{18}$

$m_{LINE} = \frac{-1}{9}$

$\frac{-1}{3x^2-3} = \frac{-1}{9}$

$3x^2 - 3 = 9$

$3x^2 - 3 - 9 = 0$

$3x^2 - 12 = 0$

$$3(x^2-4)=0$$
$$3(x+2)(x-2)=0$$
$$x=-2,\ +2$$

$f(2)$

$(2,\ ?)$

$(-2,\ ?)$

$f(-2)$

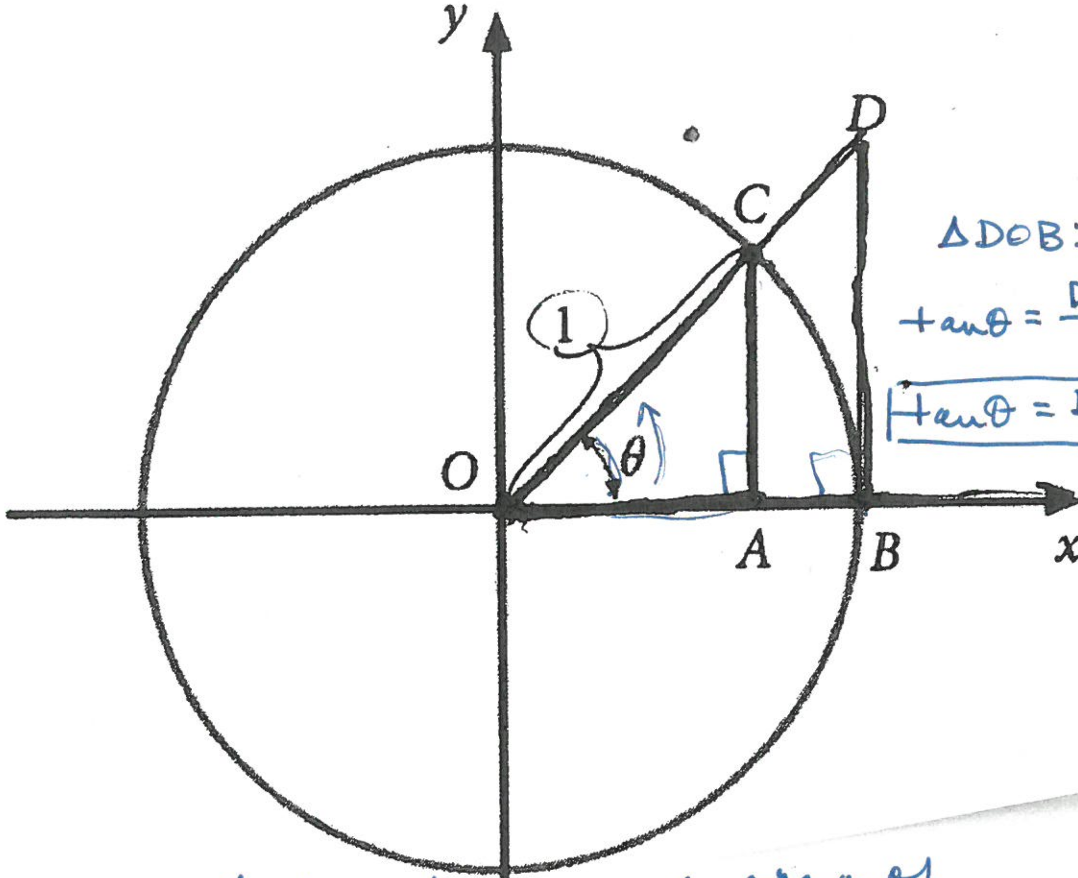
$$f(x)=x^3-3x$$
$$f(2)=2$$
$$f(-2)=-2$$

$(2,2)$ & $(-2,-2)$

① $(2,2)$ $m=-1/9$ ② $(-2,-2)$ $m=\frac{-1}{9}$

$$y-2=\frac{-1}{9}(x-2)$$
$$y-(-2)=\frac{-1}{9}(x-(-2))$$

unit circle: $r = 1$



$\triangle COA$:

$$\sin\theta = \frac{CA}{1}$$

$$\boxed{\sin\theta = CA}$$

$$\cos\theta = \frac{OA}{1}$$

$$\boxed{\cos\theta = OA}$$

$\triangle DOB$:

$$\tan\theta = \frac{DB}{1}$$

$$\boxed{\tan\theta = DB}$$

area of $\triangle COA$:

$\frac{1}{2}(\text{base})(\text{height})$

$\frac{1}{2}(OA)(AC)$

$<$ area of SECTOR COB

$\left(\frac{\theta}{2\pi}\right)(\pi \cdot r^2)$

$\frac{\theta}{\cancel{2\pi}_{2}}(\cancel{\pi} \cdot)$

$\frac{1}{2}(\theta)$

$<$ area of $\triangle DOB$

$\frac{1}{2}(\text{base})(\text{height})$

$\frac{1}{2}(OB)(BD)$

$$\frac{1}{2}(OA)(AC) < \frac{1}{2}\theta < \frac{1}{2}(OB)(BD)$$

$$\underset{\downarrow}{(OA)}\underset{\downarrow}{(AC)} < \theta < (OB)(BD)$$

$$(\cos\theta)(\sin\theta) < \theta < (1)(\tan\theta)$$

$$\cos\theta \cdot \sin\theta < \theta < \tan\theta$$

$$\frac{\cos\theta \cdot \cancel{\sin\theta}}{\cancel{\sin\theta}} < \frac{\theta}{\sin\theta} < \frac{\cancel{\sin\theta}}{\cos\theta} \cdot \frac{1}{\cancel{\sin\theta}}$$

$$\cos\theta < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$$

$$\boxed{\cos\theta} < \frac{\theta}{\sin\theta} < \boxed{\frac{1}{\cos\theta}}$$

$$\lim_{\theta \to 0}$$

$$1 < \frac{\theta}{\sin\theta} < 1$$

$$\boxed{\lim_{\theta \to 0} \frac{\theta}{\sin\theta} = 1}$$