MA141 – 005

Monday, February 4

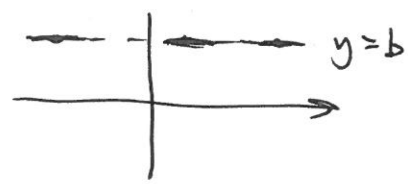
- TEST #1 (first hour of class)
- CH 2 (second hour of class)
  - power rule
  - product rule
  - quotient rule

} from VIDEOS

2.2: DERIVATIVE RULES

① DERIV OF CONSTANT:

$$\boxed{f(x) = b \qquad f'(x) = 0}$$

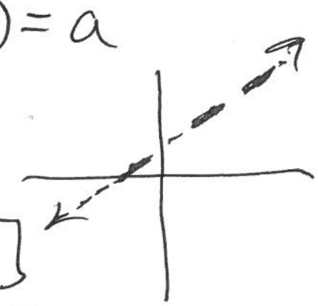


$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{b - b}{h}$$

$$= \lim_{h\to 0} \frac{0}{h} = 0$$

② $f(x) = ax + b$ (LINEAR) $\quad f'(x) = a$



$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[a(x+h)+b] - [ax+b]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{ax} + ah + \cancel{b} - \cancel{ax} - \cancel{b}}{h}$$

$$= \lim_{h\to 0} \frac{\boxed{a}\cancel{h}}{\cancel{h}} \quad (h \neq 0) = a$$

(3) CONSTANT MULTIPLE OF A FUNCTION:

$$\frac{d\,[k \cdot f(x)]}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{k \cdot f(x+h) - k \cdot f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\boxed{k}\,[f(x+h) - f(x)]}{h}$$

$$= k \cdot \boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$$\frac{d\,[k \cdot f(x)]}{dx} = k \cdot f'(x)$$

(4) ~~PRODUCT RULE~~: SUM/DIFFERENCE RULE:

$$\frac{d\,[f(x) + g(x)]}{dx} = \lim_{h \to 0} \frac{[f(x+h) + g(x+h)] - [f(x) + g(x)]}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x) + g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h}\right]$$

$$= f'(x) + g'(x)$$

5 ~~PR~~ POWER RULE:

$\boxed{f(x) = x^n}$ (n is a non-neg integer; n is a positive integer)

ex: $y = x^2$

$y = x^3$

$y = x^{11}$

$y = 14x^3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\boxed{(x+h)^n} - x^n}{h}$$

$$= \lim_{h \to 0} \frac{\left[\cancel{x^n} + \boxed{nx^{n-1} \cdot h} + \boxed{\frac{n(n-1)}{1 \cdot 2} x^{n-2} h^2} + \dots\right] - \cancel{x^n}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left[n \cdot x^{n-1} + \frac{n(n-1)}{1 \cdot 2} x^{n-2} \cdot h + (h\text{-terms})\right]}{\cancel{h}}$$

$(h \neq 0)$

$$= \lim_{h \to 0} \left[n \cdot x^{n-1} + \boxed{\cancel{\frac{n(n-1)}{1 \cdot 2} x^{n-2} \cdot h}}^{\,0} + \cancel{(h\text{-terms})}^{\,0}\right]$$

$f'(x) = n \cdot x^{n-1}$ $\qquad$ $f(x) = x^n$

POWER RULE:

1. $f(x) = x^n$ $\qquad$ $f'(x) = n \cdot x^{n-1}$
2. $\boxed{f(x) = a \cdot x^n \qquad f'(x) = a \cdot (n \cdot x^{n-1})}$

ex:

$$f(x) = \underbrace{3x^5} \underbrace{-11x^3} + \underbrace{8x^2} - \underbrace{4x} + \underbrace{3}$$

$$f'(x) = 3\cdot(5\cdot x^{5-1}) - 11(3x^{3-1}) + 8(2x^{2-1}) - 4(1\cdot x^{1-1}) + 0$$

$$f'(x) = 15x^4 - 33x^2 + 16x - 4$$

$(x+h)^5 \quad (x+h)^3$

generalizable:

ex: $f(x) = 8\sqrt{x} - \dfrac{5}{x^{-4}} + 11x^{-2/3}$

$$f(x) = 8x^{1/2} - 5x^{-4} + 11x^{-2/3}$$

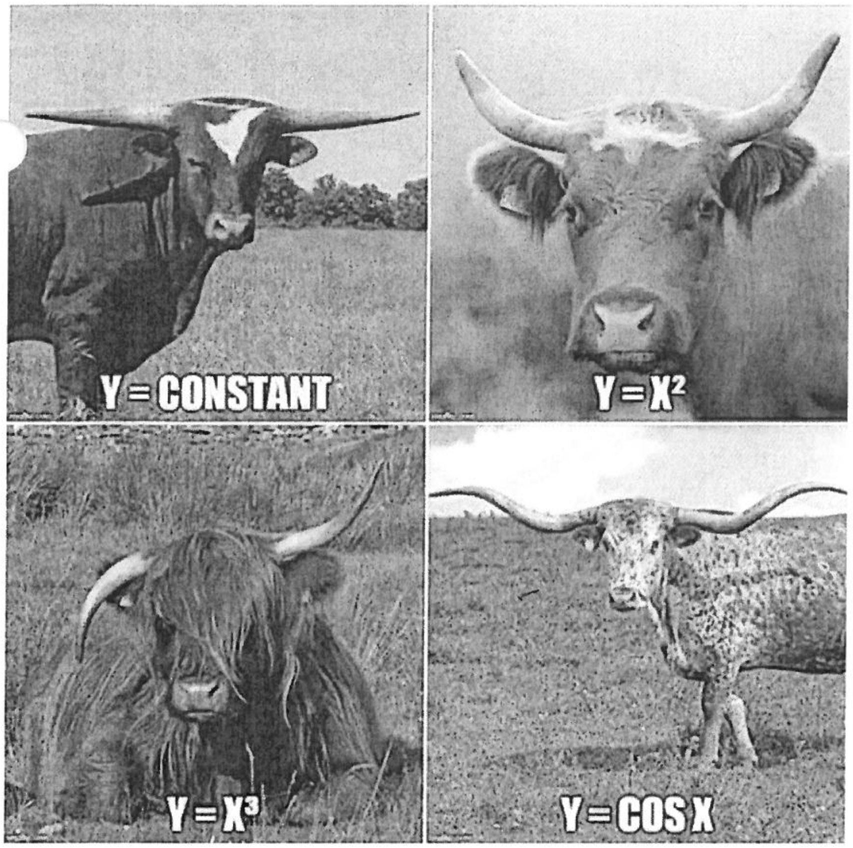
$$f'(x) = 8\left[\tfrac{1}{2}x^{1/2-1}\right] - 5\left[-4\cdot x^{-4-1}\right] + 11\left[-\tfrac{2}{3}\cdot x^{-2/3-1}\right]$$

$$\boxed{f'(x) = 4x^{-1/2} + 20x^{-5} - \frac{22}{3}x^{-5/3}}$$

$f'(1) =$

$$f'(x) = \frac{4}{\sqrt{x}} + \frac{20}{x^5} - \frac{22}{3x^{5/3}}$$

$$f'(x) = \frac{4}{\sqrt{x}} + \frac{20}{x^5} - \frac{22}{3(\sqrt[3]{x})^5}$$


Y = CONSTANT
Y = X²
Y = X³
Y = COS X