

## Summary of Convergence and Divergence Tests for Series

TEST	SERIES	CONVERGENCE OR DIVERGENCE	COMMENTS/ THINGS TO WATCH	EXAMPLE
$n$ th-term	$\sum a_n$	Diverges if $\lim_{n \rightarrow \infty} a_n \neq 0$	$= 0$ MIGHT converge; need another test	
Geometric Series	$\sum_{n=1}^{\infty} ar^{n-1}$	Diverges if $r \geq 1$ or $r \leq -1$ Converges if $-1 < r < 1$	If converges $S = \frac{a}{1-r}$ Useful with comparison test if the $n$ th term $a_n$ of a series is similar to $ar^{n-1}$	
$P$ -series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Diverges if $p \leq 1$ Converges if $p > 1$	Useful for comparison tests if the $n$ th term $a_n$ of a series is similar to $\frac{1}{n^p}$ . Called Harmonic Series when $p = 1$ .	
Integral	$\sum_{n=1}^{\infty} a_n$ where $a_n = f(n)$	Diverges if $\int_1^{\infty} f(x) dx$ diverges Converges if $\int_1^{\infty} f(x) dx$ converges	$f(x)$ must be: → positive → continuous → decreasing → able to integrate	
Comparison	$\sum a_n, \sum b_n$ $a_n > 0, b_n > 0$	Diverges: $a_n \geq b_n$ for every $n$ : if $\sum b_n$ diverges, then $\sum a_n$ diverges Converges: $a_n \leq b_n$ for every $n$ : if $\sum b_n$ converges then $\sum a_n$ converges.	$\sum b_n$ is often a geometric series or a $p$ series.	

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Limit Comparison	$\sum a_n, \sum b_n$ $a_n > 0, b_n > 0$	If $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = C > 0$ , then both series converge or both diverge. $C$ is a finite number.	To find $b_n$ , consider only the terms of $a_n$ that have the greatest effect in the magnitude. If $C = 0$ and $b_n$ converges, then $a_n$ converges.	
Alternating Series	$\sum (-1)^n a_n$ $a_n > 0$	2 conditions: 1. $\lim_{n \rightarrow \infty} a_n = 0$ 2. $a_{n+1} \leq a_n$	Must Alternate!	
Absolute Convergence	$\sum a_n$ (not alternating series, but some terms negative)	If $\sum  a_n $ converges, then $\sum a_n$ converges	*Absolute convergence implies convergence *If diverges, might converge-need another test	
Ratio Test	$\sum a_n$	If $\lim_{n \rightarrow \infty} \left  \frac{a_{n+1}}{a_n} \right  = L$ or $\infty$ then: Diverges if $L > 1$ or $\infty$ Converges if $L < 1$ Inconclusive $L = 1$	Useful if $a_n$ involves factorials or $n$ th powers. If $a_n > 0$ for every $n$ , the absolute value sign may be disregarded.	