MA141 – 005

Wednesday, January 30

- finish ch 1 and 2.1
- test review

MON, FEB. 4th, TEST #1
(ch 0; ch 1; 2.1)

CH 1:

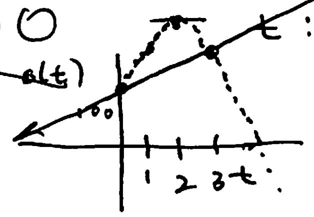
instantaneous velocity:

$s(t) = -16t^2 + 64t + 100$

↑ dist; ht; pos.

s(t): feet

t: seconds



① $t=0 \quad s=?$ $\quad s(0) = -16(0)^2 + 64(0) + 100 = 100$ ft.

$t=1 \quad s=?$ $\quad s(1) = -16(1)^2 + 64(1) + 100 = 148$ ft.

$-16 \quad +164$

$t=2 \quad s=?$ $\quad s(2) = -16(2)^2 + 64(2) + 100 = 164$ ft

$-64 \quad +128 \quad +100$

$t=3 \quad s=?$ $\quad s(3) = -16(3)^2 + 64(3) + 100 = 148$ ft

$-144 \quad +192 \quad +100$

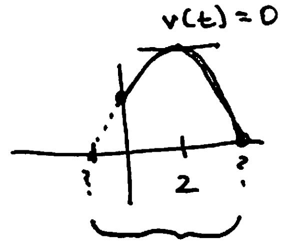
② average rate of change
= average velocity
from $t=0$ $t=3$

slope of secant line

$$\boxed{\frac{s(3)-s(0)}{3-0}} = \frac{148\text{ ft} - 100\text{ ft}}{3\text{ sec}} = \frac{48\text{ ft}}{3\text{ sec}} = 16\,\frac{\text{ft}}{\text{sec}}$$

(3) instantaneous velocity
= velocity

(at a certain t-value)



$s(t) = -16t^2 + 64t + 100$

$$s'(t) = \boxed{v(t)} = \lim_{h \to 0} \left[\frac{s(t+h) - s(t)}{h} \right]$$

$$= \lim_{h \to 0} \frac{[-16(t+h)^2 + 64(t+h) + 100] - [-16t^2 + 64t + 100]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-16t^2} - 32th - 16h^2 + \cancel{64t} + 64h + \cancel{100} + \cancel{16t^2} - \cancel{64t} - \cancel{100}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-32t + 64 - 16h)}{\cancel{h}} \quad (h \neq 0)$$

$$= \lim_{h \to 0} (-32t \cancel{-16h} + 64) = -32t + 64$$

$$v(t) = -32t + 64$$

- inst. vel. at $t = 0$

$v(0) = -32(0) + 64 = 64$ ft/sec

$v(1) = -32(1) + 64 = 32$ ft/sec

$v(2) = -32(2) + 64 = 0$ $\quad \dfrac{-32 \text{ ft/sec}}{\text{sec}} = -32 \text{ ft/sec}^2$

when does the object STRIKE THE GROUND

$s(t) = 0$

$-16t^2 + 64t + 100 = 0$

$$t = \frac{-64 \pm \sqrt{(64)^2 - 4(-16)(100)}}{2(-16)}$$

$$t = \frac{-64 \pm \sqrt{64 + 6400}}{-32}$$

$t_1 \to t_1 \approx 5.2$

$t_2 \to t_2 \approx \cancel{-}$

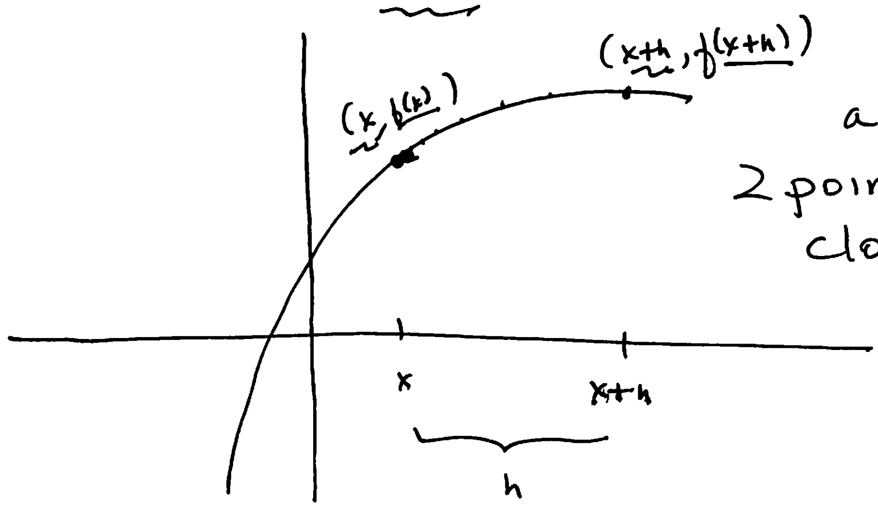
Chapter 2:

2.1:

DEF. OF DERIV:

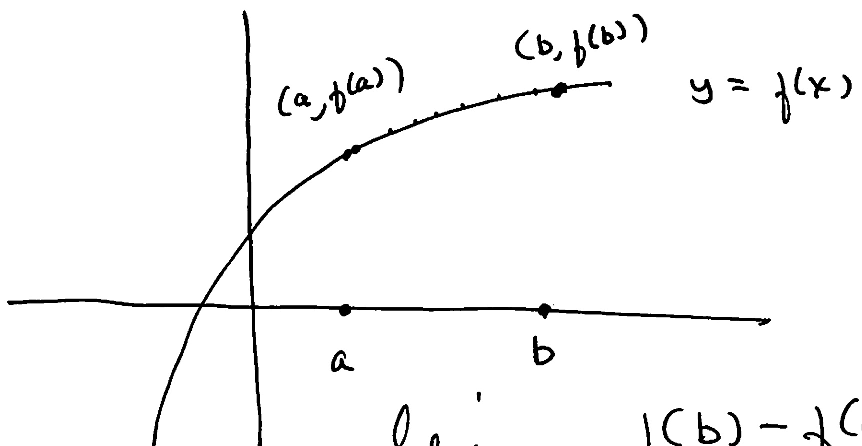
$f(x) = $ ~~~~

$$f'(x) = m_{TAN} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



as $h \to 0$
2 points get closer together

ALT. DEF. OF DERIVATIVE:



$$f'(x) = \lim_{b \to a} \frac{f(b) - f(a)}{b - a}$$

$$V(t) = 50t^2$$

$V(t)$: gallons
t: minutes

(1) $t=1$ $\quad t=2$

average rate of change

$$\frac{V(2)-V(1)}{2-1} = \frac{50(2)^2 - 50(1)^2}{1} \frac{\text{gal}}{\text{min}}$$

$$150 \frac{\text{gal}}{\text{min}}$$

(2) inst. rate of change:

$$V'(t) = \lim_{h \to 0} \frac{V(t+h) - V(t)}{h}$$

$$= \lim_{h \to 0} \frac{50(t+h)^2 - 50t^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{50t^2} + 100th + 50h^2 - \cancel{50t^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(100t + 50h)}{\cancel{h}} \quad (h \neq 0)$$

$$= \lim_{h \to 0} (100t + \cancel{50h}) = 100t \frac{\text{gal}}{\text{min}}$$

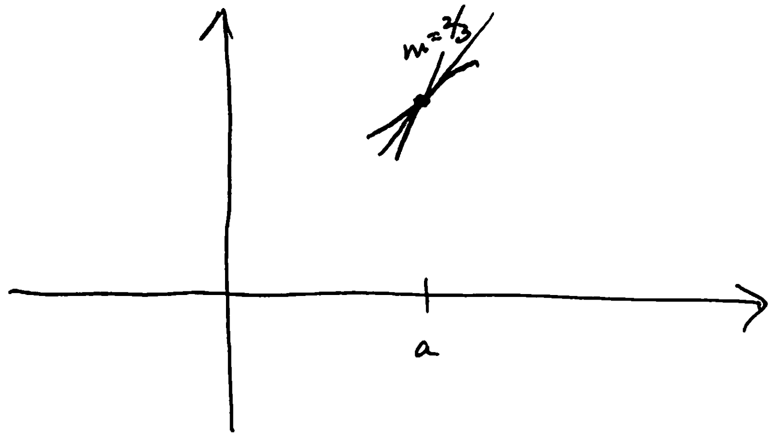
$t=1$

$$V'(1) = 100(1) = 100 \frac{\text{gal}}{\text{min}}$$

if ~~~, then ~~~

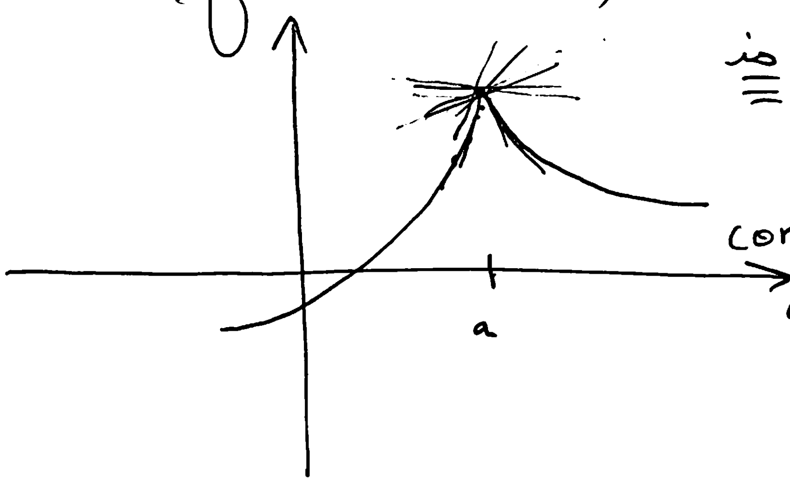
if $f(x)$ is DIFF, then $f(x)$ is CONTIN

DIFF $\longrightarrow$ CONTIN.



CONVERSE:

if CONTIN, then DIFF.



is CONTIN, is NOT DIFF.

corner; cusp; abrupt change in dir.

using ALT DEF OF DERIV:

$$V(t) = 50t^2 \qquad t=1 \qquad (1, V(1)) \quad (t, V(t))$$

$$V'(t) = \lim_{t \to 1} \frac{V(t) - V(1)}{t-1}$$

$$= \lim_{t \to 1} \frac{50t^2 - 50}{t-1}$$

$$= \lim_{t \to 1} \frac{50(t^2-1)}{t-1}$$

$$= \lim_{t \to 1} \frac{50(t+1)\cancel{(t-1)}}{\cancel{t-1}} \qquad \underline{t \neq 1}$$

$$= \lim_{t \to 1} 50(t+1) = 100 \text{ gal/min}$$

$=$

NOTATIONS FOR DERIV:

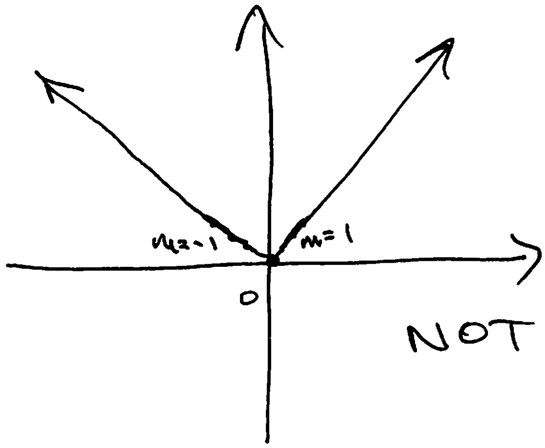
$\cancel{f'(x)}$; $\frac{dy}{dx}$; y' ; $D_x y$; $\left.\frac{dy}{dx}\right|_{x=1}$

"deriv of y with respect to x"

$f'(2)$

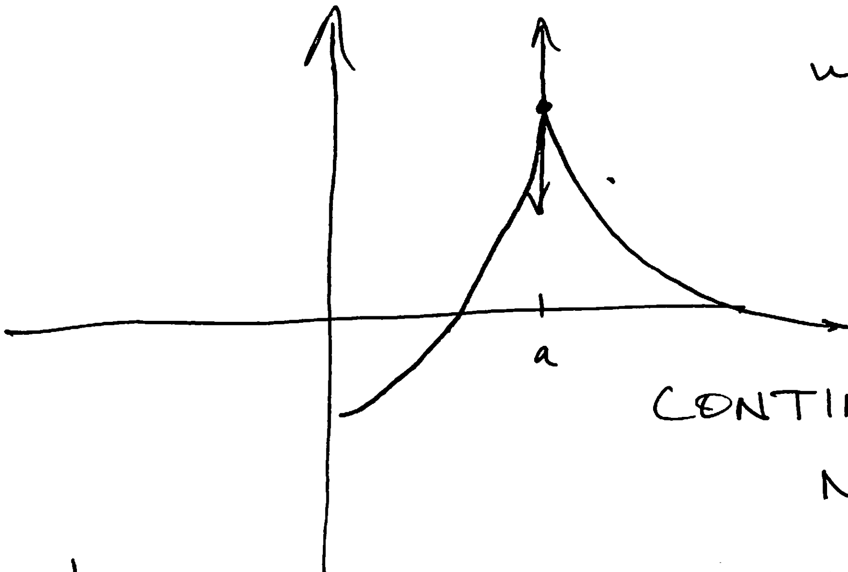
$\frac{\Delta y}{\Delta x}$

$f''(x)$ or $f'''(x)$ or $f^{(4)}(x)$



$f(x) = |x|$

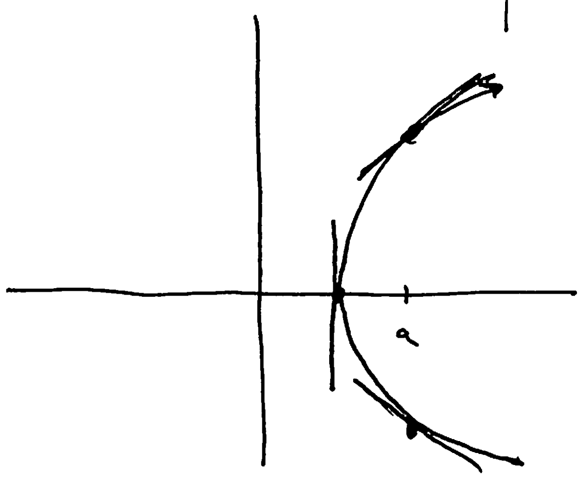
NOT DIFF. AT $x = 0$

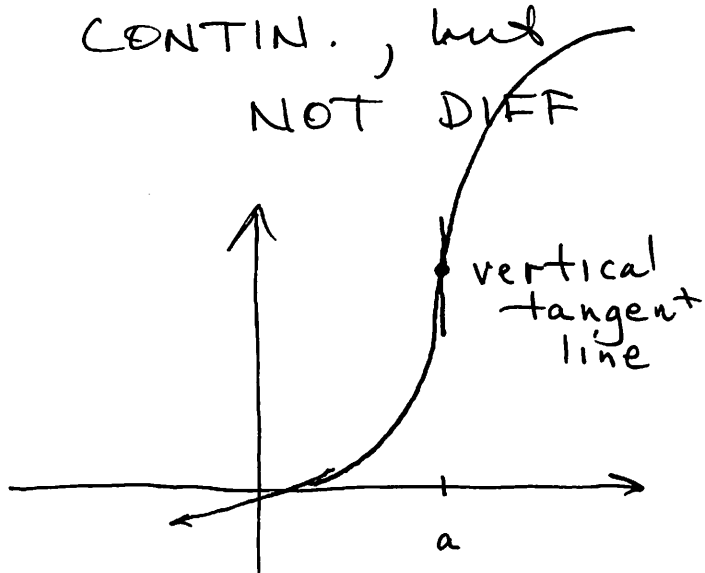


vertical tangent

m_{TAN} = <u>undef.</u>

CONTIN., but NOT DIFF

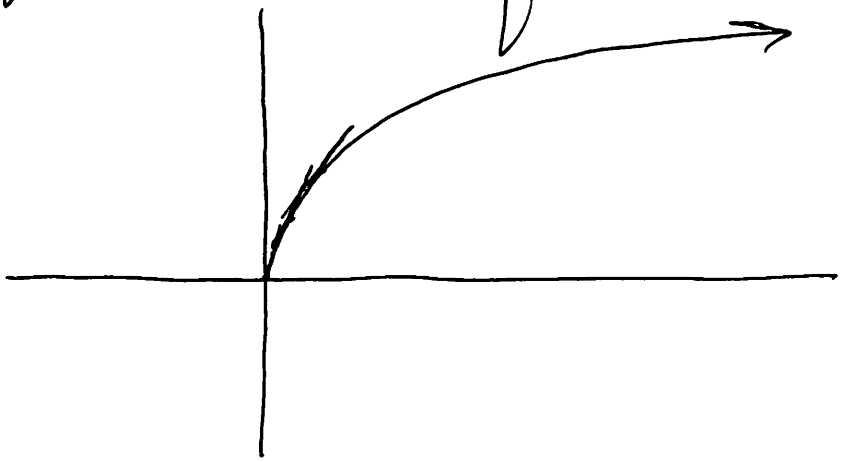




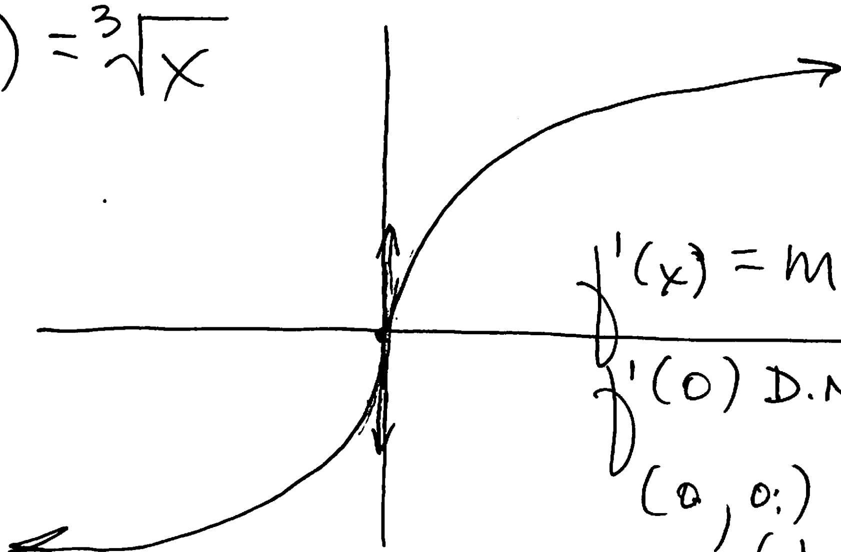
$f'(a)$ D.N.E.

$f(x) = \sqrt{x}$

$f'(0)$ DNE



$f(x) = \sqrt[3]{x}$

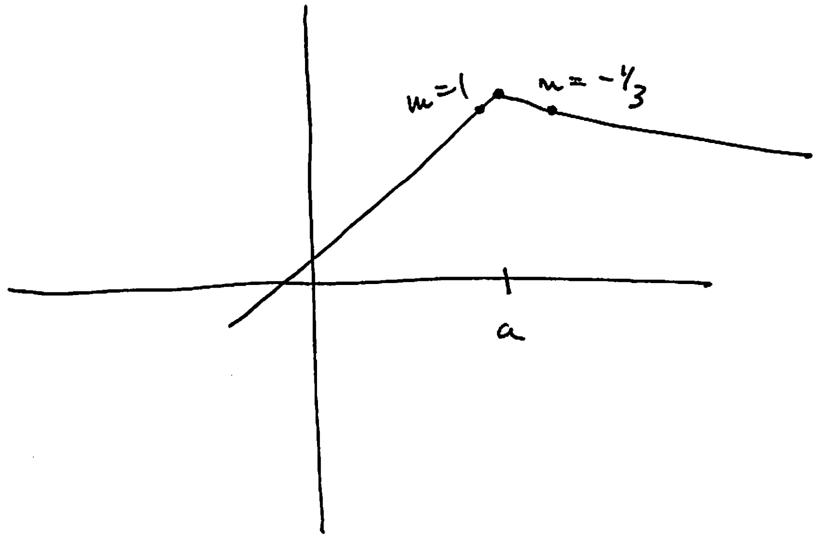


Show that $f(x)$ is NOT DIFF at $x = 0$ (ALT. DEF OF DERIV.)

$$f'(x) = \lim_{h \to 0} \frac{h^{1/3} - 0}{h - 0}$$

$$f'(x) = \lim_{h \to 0} \frac{h^{1/3}}{h^1} = \lim_{h \to 0} h^{-2/3}$$

$$= \lim_{h \to 0} \frac{1}{h^{2/3}} \text{ DNE}$$



$f(x) =$ $\qquad$ $g(x) =$

$(f \circ g)(x) = f(g(x)) =$

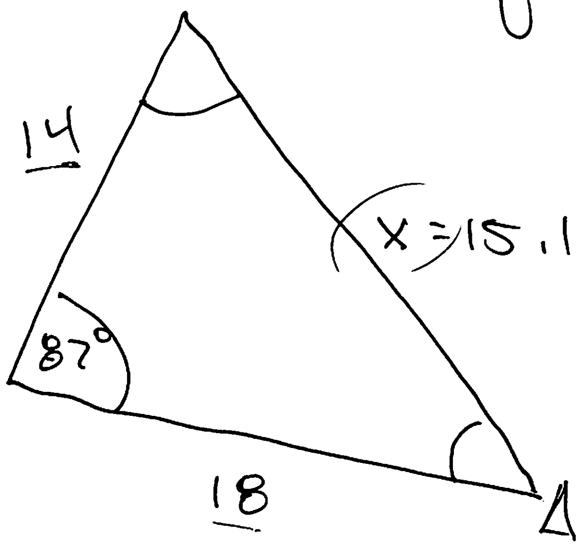
$=$

$=$

$=$

(Law of Cosines

SAS

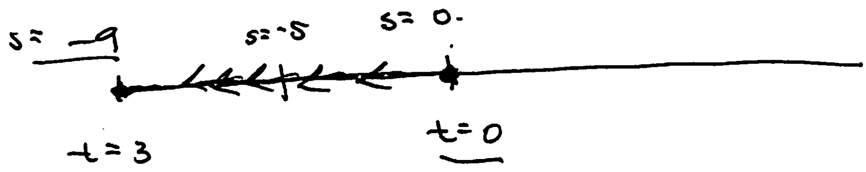


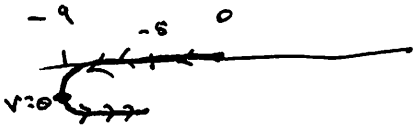
law of cosines:

$$x^2 = 14^2 + 18^2 - 2(14)(18)\cos 87^\circ$$

$$\frac{\sin 87}{15.1} = \frac{\sin A}{14}$$

$$\frac{\cancel{(a-b)}\,-1}{\cancel{(b-a)}\;\;1} \qquad \frac{7-4}{4-7}$$





$[0, 5]$

$v(t) = 2t - 6$

$0 = 2t - 6$

$6 = 2t$

$\underline{3 = t}$

$s(0) = \underline{0}$

$s(3) = \underline{-9}$

$s(5) = \underline{-5}$

$s(t) = t^2 - 6t$

$s(0) = 0^2 - 6(0) = 0$

$s(3) = 3^2 - 6 \cdot 3 = -9$

$s(5) = 5^2 - 6 \cdot 5 = -5$