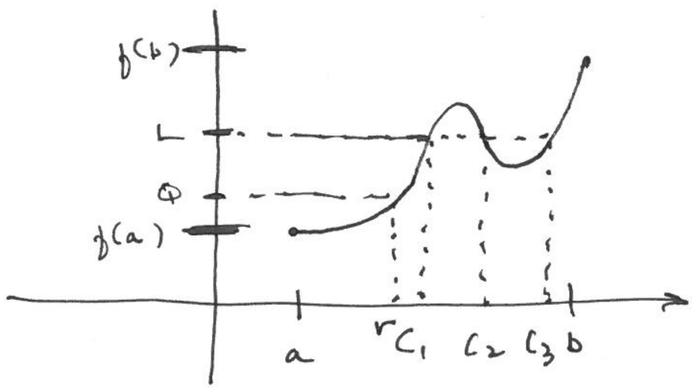
MA141-005

Monday, January 28

- TEST #1: MON, FEB 4 (CH 0; CH 1; 2.1)
- TODAY:
  - more with I.V.T.
  - average rate of change
  - instantaneous rate of change

INTERMEDIATE VALUE THEOREM:



f(x) on [a, b]

must be continuous on [a, b]

the function values take on every value from f(a) to f(b) as x goes from a to b.

(AT LEAST ONE VALUE C IN [a, b] SUCH THAT $f(c) = L$)

show that $f(x)$ has a root (zero) in $[1,2]$

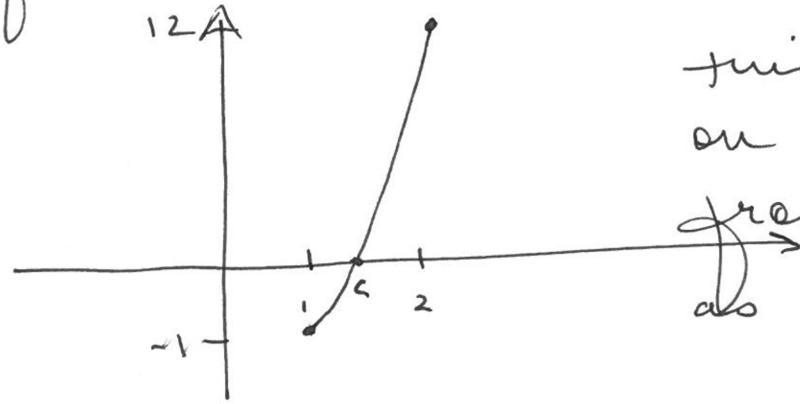
$$f(x) = 4\boxed{x^3} - 6\boxed{x^2} + 3\boxed{x^1} - 2\cdot(x^0)$$

(1) ¿continuous?

yes, it is a polynomial (exponents are NON-NEG INTEGERS)

(2) $f(1) = 4(1)^3 - 6(1)^2 + 3(1) - 2 = 4 - 6 + 3 - 2 = -1$

$f(2) = 4(2)^3 - 6(2)^2 + 3(2) - 2 = 32 - 24 + 6 - 2 = 12$



this $f(x)$ takes on ALL values from -1 to 12 as x goes from 1 to 2.

(there is at least one number c in $[1,2]$ such that $f(c) = 0$ by I.V.T.)

$f(x) = \sin\left(\frac{\pi \cdot x}{4}\right)$ $\quad g(x) = x^2 - x$

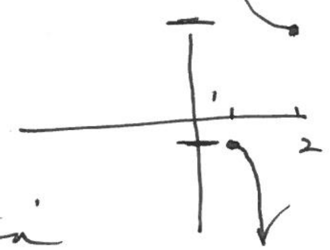
$[1, 2]$

show that $f(x) = g(x)$ at least once in the interval $[1, 2]$.

$$\underbrace{h(x) = \sin\left(\frac{\pi \cdot x}{4}\right) - (x^2 - x)}$$

if $f(x) = g(x)$, then $h(x) = 0$

show that $h(x)$ has a root in $[1, 2]$



① ¿ continuous?

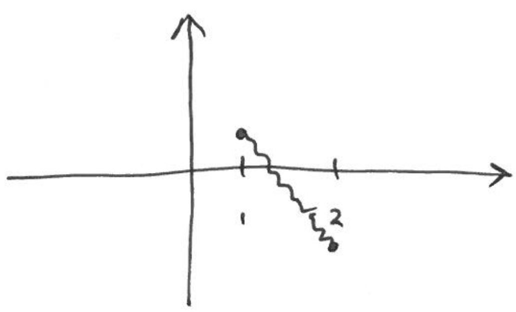
$x^2 - x$ is polynom ∴ contin

$\sin\left(\frac{\pi \cdot x}{4}\right)$ contin ($\sin x$, $\cos x$)

sum/diff. of contin functions is CONTINUOUS.

② $h(1) = \sin\left(\frac{\pi \cdot 1}{4}\right) - (1^2 - 1) = \frac{\sqrt{2}}{2} \approx (.707)$

$h(2) = \sin\left(\frac{\pi \cdot 2}{4}\right) - (2^2 - 2) = 1 - (2) = -1$

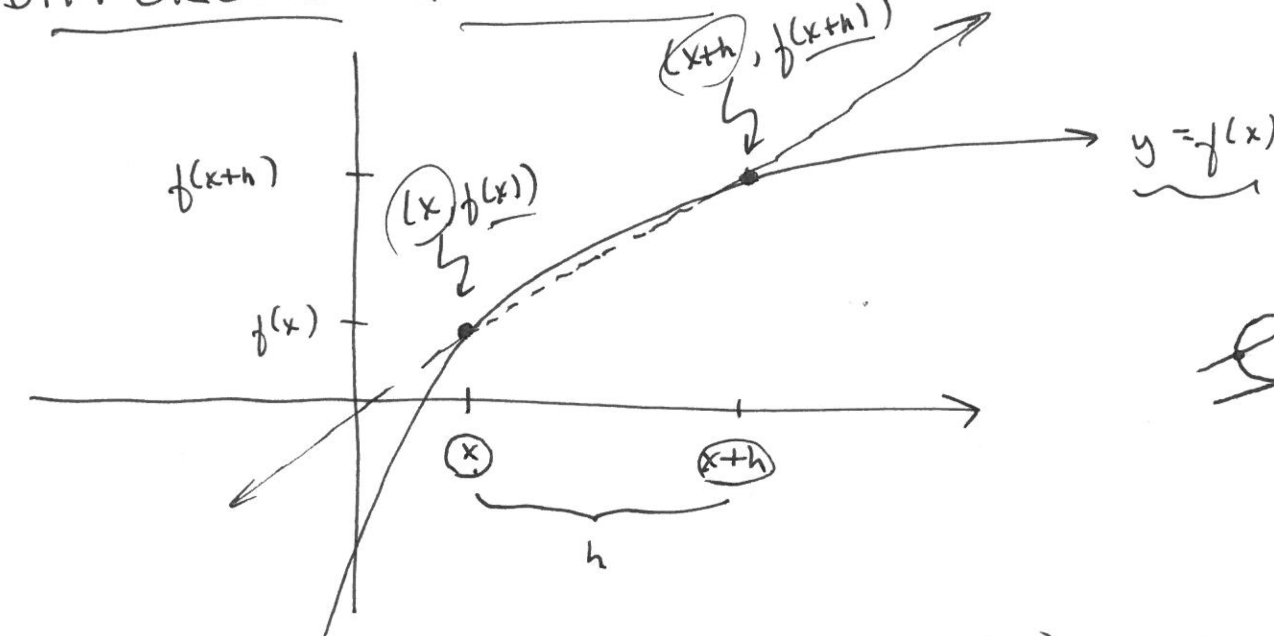


by I.V.T.
there is a c in $[1, 2]$ such that $h(c) = 0$

$\therefore f(x) = g(x)$

AVERAGE RATE OF CHANGE

- SLOPE OF THE SECANT LINE (m_{sec})
- DIFFERENCE QUOTIENT



$$m_{SEC} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

---

ex: $f(x) = 5x^2 - 8x + 11$

x: ~~min~~ hr
$f(x)$: miles

find the m_{sec}:

$$m_{SEC} = \frac{f(x+h) - f(x)}{h} \begin{matrix} \to \text{miles} \\ \to \text{hours} \end{matrix}$$

$$= \frac{\left[5(x+h)^2 - 8(x+h) + 11\right] - \left[5x^2 - 8x + 11\right]}{h}$$

$$= \frac{5(x^2 + 2xh + h^2) - 8x - 8h + 11 - 5x^2 + 8x - 11}{h}$$

$$= \frac{\cancel{5x^2} + 10xh + 5h^2 - \cancel{8x} - 8h + \cancel{11} - \cancel{5x^2} + \cancel{8x} - \cancel{11}}{h}$$

$$= \frac{h(10x + 5h - 8)}{h} \quad (h \neq 0)$$

$$= \frac{\cancel{h}(10x+5h-8)}{\cancel{h}} \quad (h \neq 0)$$

$$\boxed{m_{SEC} = 10x + 5h - 8}$$

x: initial x-value

h: change in x

ANY TWO POINTS

$(\boxed{1}, 8)$

$h=2$

$(\boxed{3}, 32)$

$x=1$
$h=2$

$$m_{SEC} = 10(1) + 5(2) - 8$$
$$= 10 + 10 - 8$$
$$= 12$$

$$m = \frac{32-8}{3-1} = \frac{24}{2} = 12$$

find the average rate of change for $f(x) = 2x^2 + 8x + 5$ from ...

① $x = 1$ to $x = 3$ $\longrightarrow \dfrac{f(3) - f(1)}{3-1}$

② $x = 1$ to $x = 2$ $\longrightarrow \dfrac{f(2) - f(1)}{2-1}$

③ $x = 1$ to $x = 1.5$ $\longrightarrow \dfrac{f(1.5) - f(1)}{1.5-1}$

$$f(x) = \frac{2x+1}{5x+3}$$

$$\frac{5\cdot 1}{5\cdot 3} - \frac{1\cdot 3}{5\cdot 3}$$

$$\frac{5}{15} - \frac{3}{15}$$

$$\frac{5-3}{15}$$

$$m_{SEC} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{\boxed{\dfrac{2(x+h)+1}{5(x+h)+3}} - \boxed{\dfrac{2x+1}{5x+3}}}{h}$$

$$= \left[\frac{(2(x+h)+1)(5x+3)}{(5(x+h)+3)(5x+3)} - \frac{(2x+1)(5(x+h)+3)}{(5x+3)(5(x+h)+3)}\right]\cdot\frac{1}{h}$$

$$= \frac{(2(x+h)+1)(5x+3) - (2x+1)(5(x+h)+3)}{(5(x+h)+3)(5x+3)\cdot h}$$

$$= \frac{(2x+2h+1)(5x+3) - (2x+1)(5x+5h+3)}{(5(x+h)+3)\cdot(5x+3)\cdot h}$$

$$= \frac{(10x^2+10xh+5x+6x+6h+3) - (10x^2+10xh+6x+5x+5h+3)}{(5(x+h)+3)\cdot(5x+3)\cdot h}$$

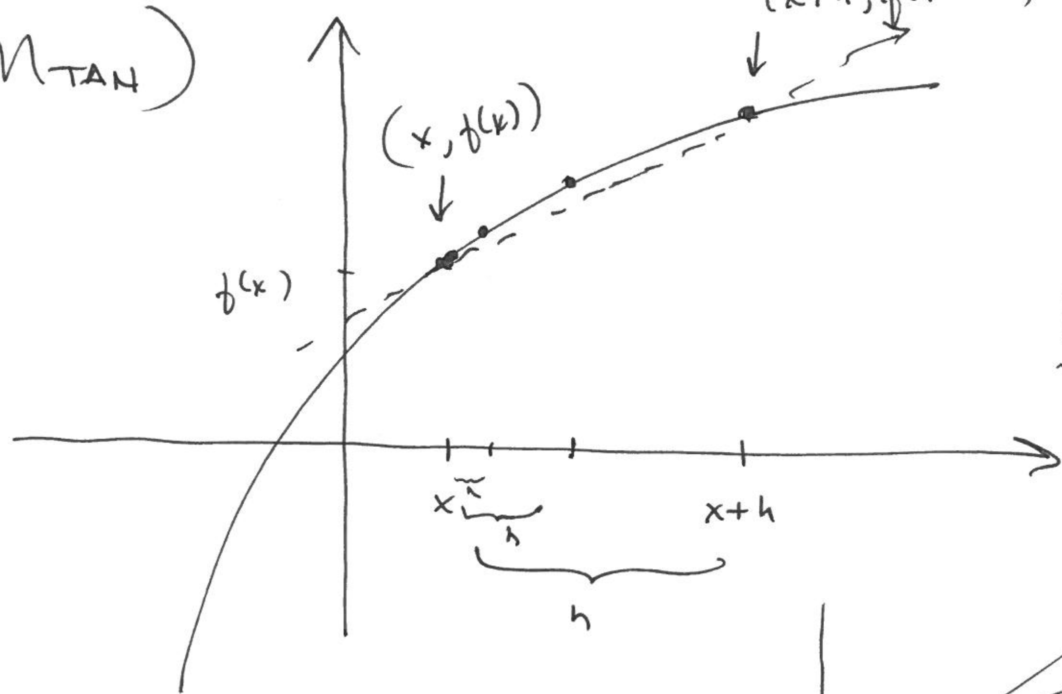
$$= \frac{\cancel{10x^2}+\cancel{10xh}+\cancel{5x}+\cancel{6x}+6h+\cancel{3}-\cancel{10x^2}-\cancel{10xh}-\cancel{6x}-\cancel{5x}-5h-\cancel{3}}{(5(x+h)+3)\cdot(5x+3)\cdot h}$$

$$= \frac{\cancel{h}^{\,1}}{(5(x+h)+3)\cdot(5x+3)\cdot\cancel{h}_{\,1}} = \boxed{\frac{1}{(5(x+h)+3)(5x+3)} = m_{SEC}}$$

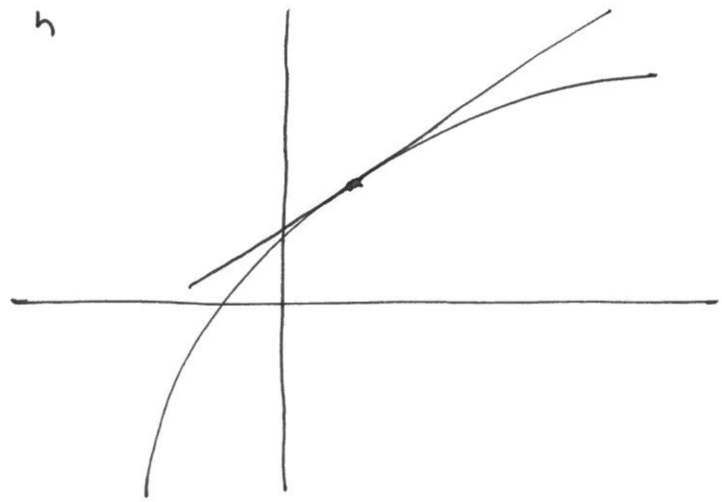
$(h \neq 0)$

$\boxed{\text{INSTANT}}$ANEOUS RATE OF CHANGE

(m_{TAN})



$h \to 0$



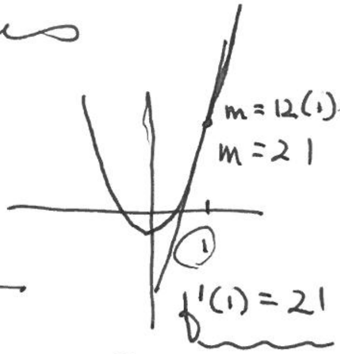
$$\lim_{h \to 0} (m_{SEC}) = m_{TAN} = \text{INST. RATE OF CHANGE}$$

$$\lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right) = m_{TAN} = f'(x)$$

f "prime" of x

DERIVATIVE

find the instantaneous rate of change



on $f(x) = 6x^2 + 9x - 1.$

(m_{TAN} ; $f'(x)$; the DERIVATIVE)

$$\lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right) = m_{TAN} = f'(x)$$

the DEF. OF DERIV.

$$\lim_{h \to 0} \left(\frac{[6(x+h)^2 + 9(x+h) - 1] - [6x^2 + 9x - 1]}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{\cancel{6x^2} + 12xh + 6h^2 + \cancel{9x} + 9h - \cancel{1} - \cancel{6x^2} - \cancel{9x} + \cancel{1}}{h} \right)$$

$$= \lim_{h \to 0} \frac{\cancel{h}(12x + 6h + 9)}{\cancel{h}} \quad (h \neq 0)$$

$$= \lim_{h \to 0} (12x + \cancel{6h} + 9) = 12x + 9 = m_{TAN}$$

slope of the tangent at ANY point on the orig $f(x)$

$$f(x) = \frac{2x+1}{5x+3}$$

$$m_{TAN} = f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

$$= \lim_{h \to 0} \left[\frac{\frac{(2(x+h)+1)}{(5(x+h)+3)} - \frac{(2x+1)}{(5x+3)}}{h} \right]$$

already done, (earlier today)

$$\vdots$$

$$= \lim_{h \to 0} \left[\frac{1}{[5(x+\cancel{h}^{0})+3] \cdot [5x+3]} \right]$$

$$= \left[\frac{1}{(5x+3)(5x+3)} \right]$$

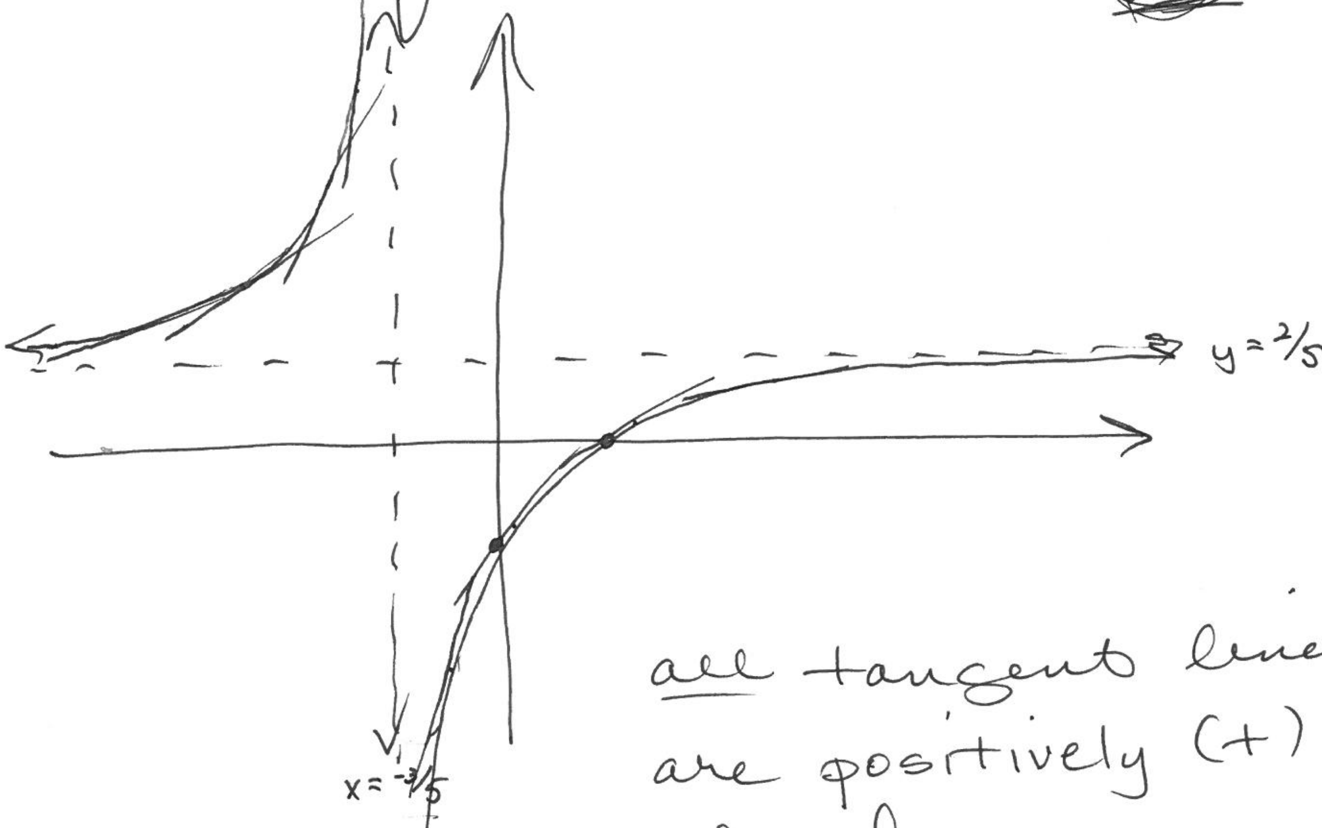
$$m_{TAN} = \frac{1}{(5x+3)^2} = f'(x)$$

$$\boxed{f(x) = \frac{2x+1}{5x+3}}$$

V.A.: $x = -3/5$

H.A.: $y = 2/5$

$$\underline{m_{TAN}} = \boxed{f'(x) = \frac{1}{(5x+3)^2}} = \frac{+}{+} \lim_{x \to \infty} \frac{2x+1}{5x+3} = +$$



<u>all</u> tangent lines are positively (+) sloped