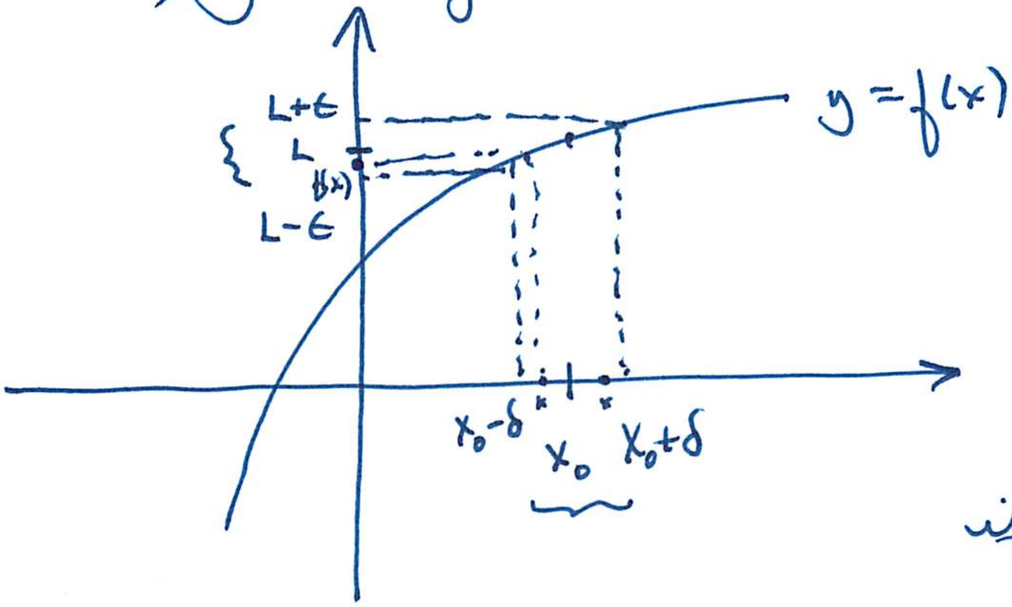
Wednesday, January 23



if ..... GIVEN

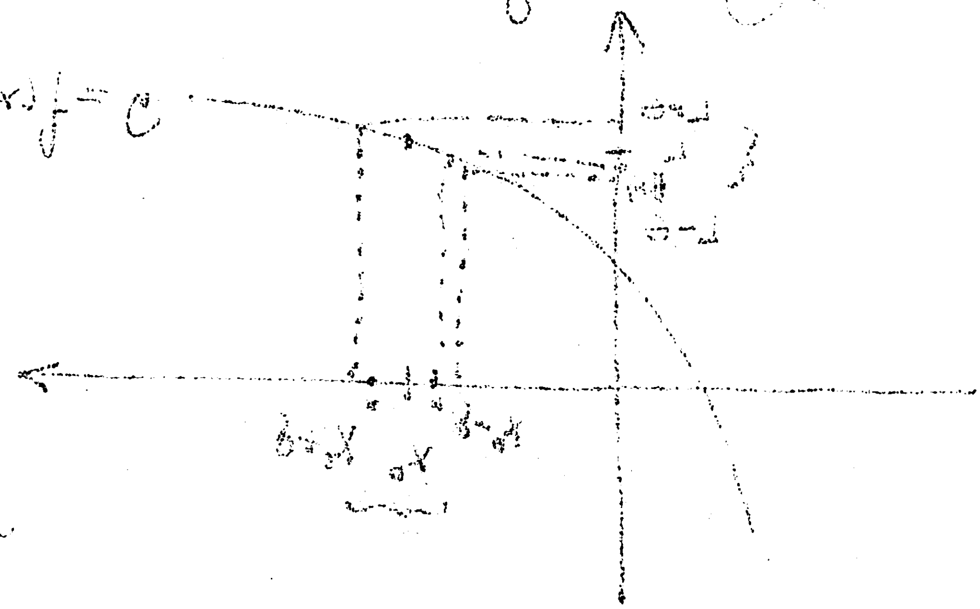
then ..... TO PROVE

$$\lim_{x \to x_0} f(x) = L$$

if $|x - x_0| < \delta$ (puts us in the δ-neighborhood)

then $|f(x) - L| < \epsilon$

$y = f(x)$



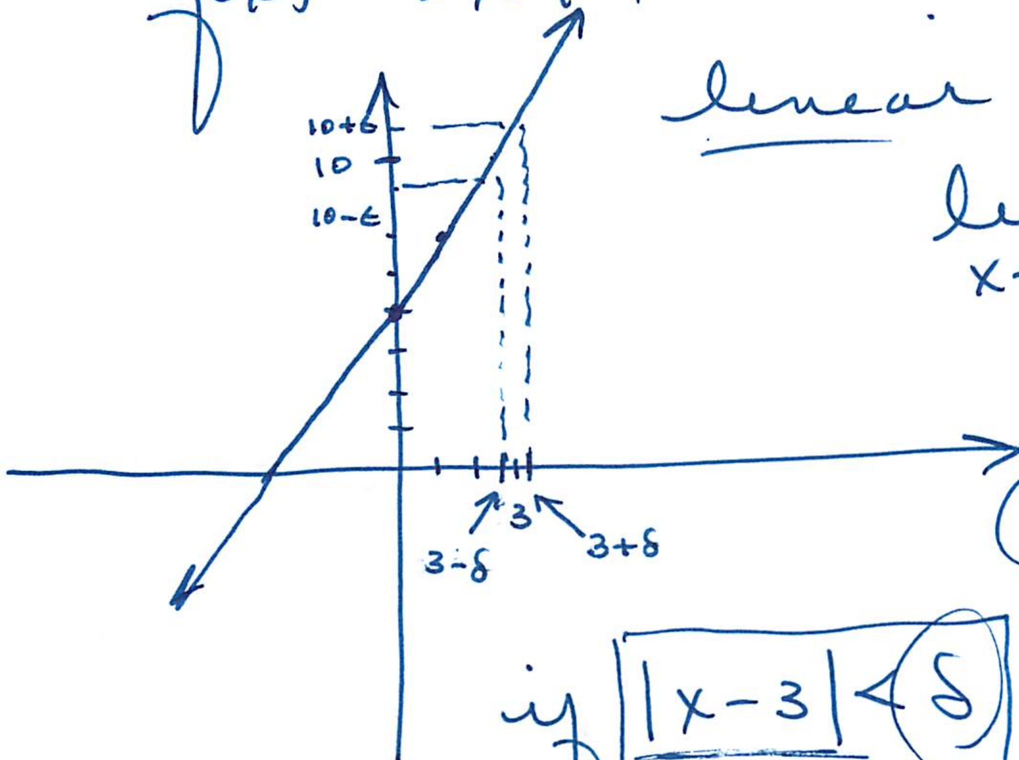
$$\lim_{x \to x_0} f(x) = L$$

$0 < |x - x_0| < \delta$

$|f(x) - L| < \varepsilon$

(2)

$f(x) = 2x+4$



linear

$$\lim_{x \to 3} (2x+4) = 10$$

$\delta = \epsilon/2$

$(2\delta = \epsilon)$

if $\boxed{|x-3| < \delta}$, then

$|f(x) - 10| < \epsilon$

---

not proof .... working backwards from what we want (relation between δ & ϵ)

$|f(x) - L| < \epsilon$

$|(2x+4) - 10| < \epsilon$

$|2x-6| < \epsilon$

$|2(x-3)| < \epsilon$

$2|x-3| < \epsilon$

$|x-3| < \frac{\epsilon}{2}$ $\boxed{\text{choose } \delta = \epsilon/2}$

(3)

is the proof: $\quad x_0 = 3$

if $|x-3| < \delta$

(choose $\delta = \epsilon/2$)

$|x-3| < \epsilon/2$

$2|x-3| < \epsilon$

$|2(x-3)| < \epsilon$

$|2x-6| < \epsilon$

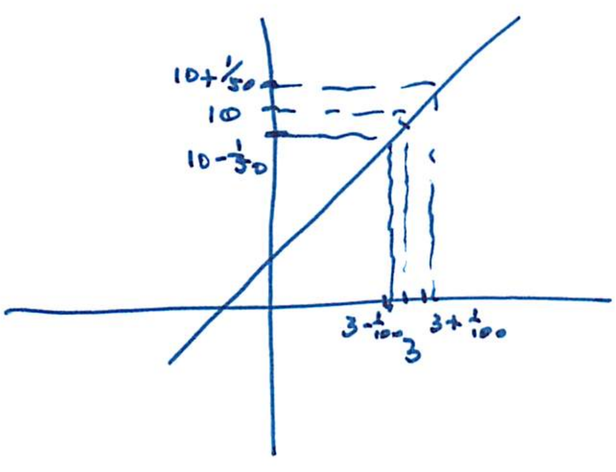
$|(2x+4)-10| < \epsilon$

then... $|f(x) - L| < \epsilon$

prove??

$|f(x) - L| < \epsilon$

$(2x+4) \quad -10$



$\delta = \frac{1}{100}$

$\epsilon = \frac{1}{50} \quad \left(\frac{2}{100}\right)$

$$\lim_{x \to 2} (3x^2 + 7x - 5) = 3(4) + 7(2) - 5$$

$$\lim_{x \to 2} 3x^2 + \lim_{x \to 2} 7x - \lim_{x \to 2} 5$$

$$3 \lim_{x \to 2} x^2 + 7 \lim_{x \to 2} x - \lim_{x \to 2} 5$$

$$3(4) \quad + 7(2) \quad - 5$$

$$12 \quad + 14 \quad - 5$$

$$= 21$$

polynomial: (CONTINUOUS)

$$f(x) = \underline{a_n} \boxed{x^n} + \underline{a_{n-1}} \boxed{x^{n-1}} + \underline{a_{n-2}} \boxed{x^{n-2}} + \dots + \underline{a_2} \boxed{x^2} + \underline{a_1} \boxed{x^1} + a_0$$

exponents $(n, n-1, n-2, \dots, 2, 1, 0)$

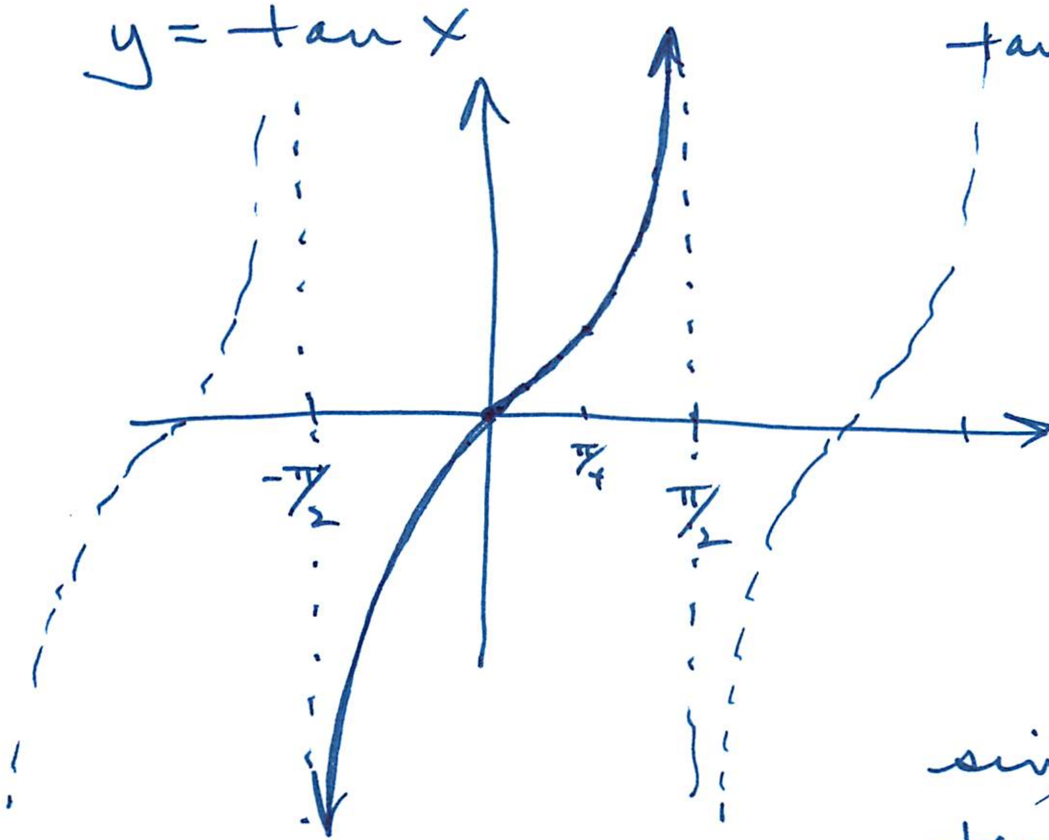
<u>NON</u>-NEGATIVE INTEGERS

ex:

$$f(x) = 81x^{29} - 17x^{18} + 43x^9 - 11x^3 + \frac{17}{3}x^2 - x + 4$$

$y = \tan x$

$\tan x = \dfrac{\sin x}{\cos x}$



~~$\sin x = 31$~~

$\tan x = 3107$

INVERSE: ① sw. x & y
② solve for y

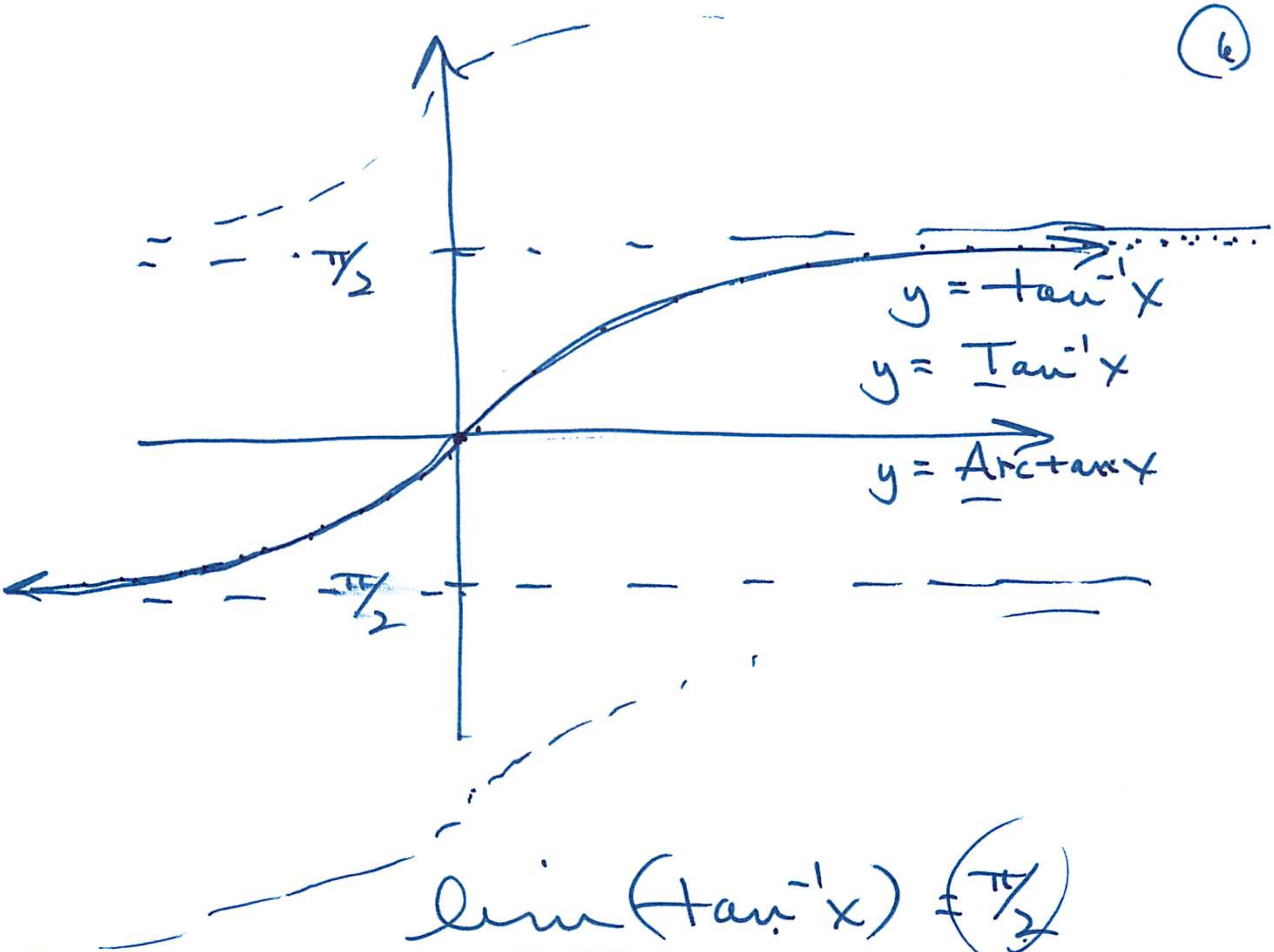
$x = \tan y$

y $\boxed{\text{is}}$ the angle whose tangent is x

$y = \tan^{-1} x$

$(y = \arctan x)$

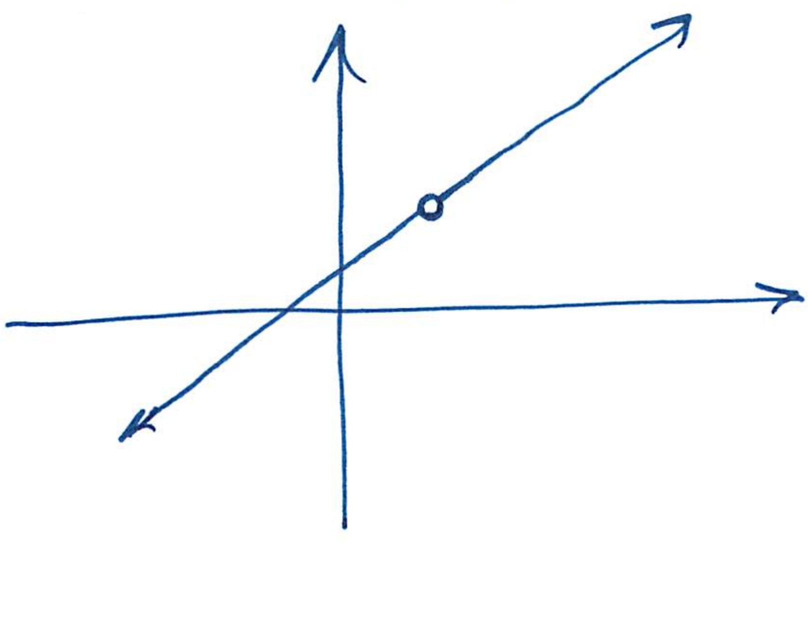
6

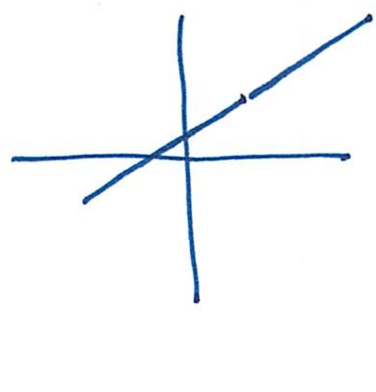


$$\lim_{x\to\infty}(\tan^{-1}x) = \frac{\pi}{2}$$

$$\lim_{x\to-\infty}(\tan^{-1}x) = -\frac{\pi}{2}$$

CONTINUITY:

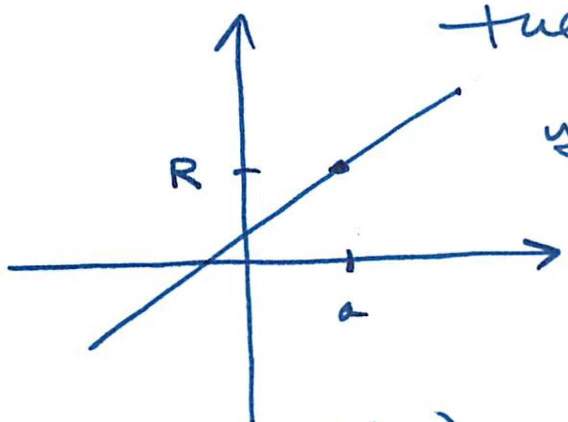




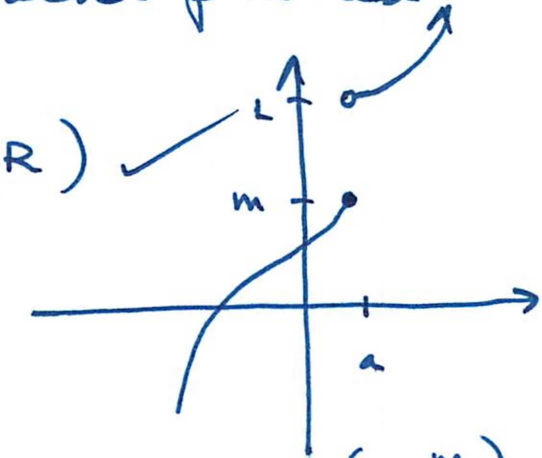
¿ contin at $x = a$?

① ¿ $f(a)$ exists?

"is there a point plotted there?"

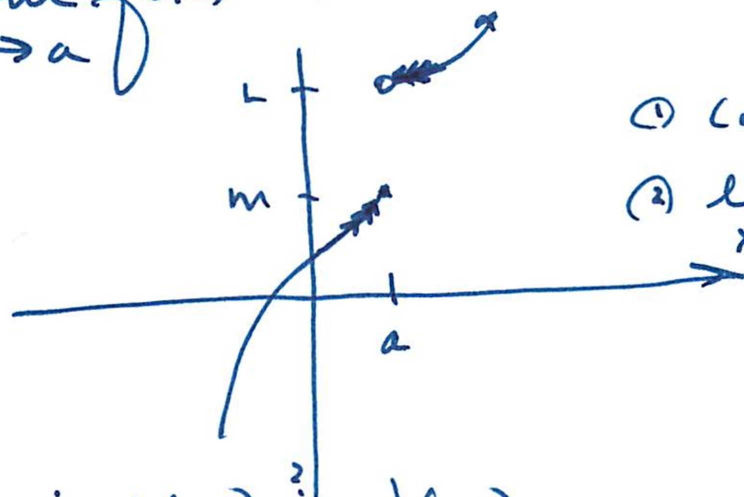


yes, (a, R) ✓



(a, m)

② ¿ $\lim_{x \to a} f(x)$ exists?

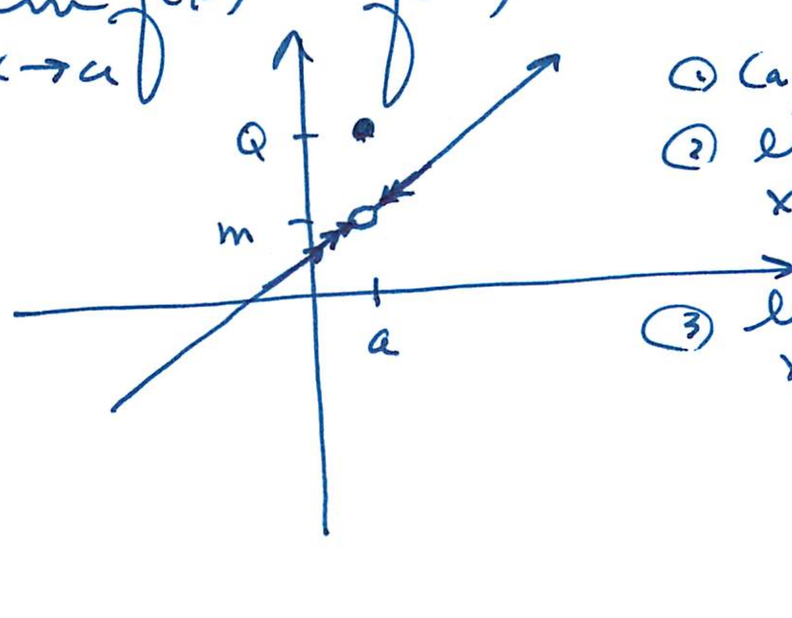


① (a, m) ✓

② $\lim_{x \to a} f(x) = \text{D.N.E.}$

∴ DISCON

③ $\lim_{x \to a} f(x) \stackrel{?}{=} f(a)$



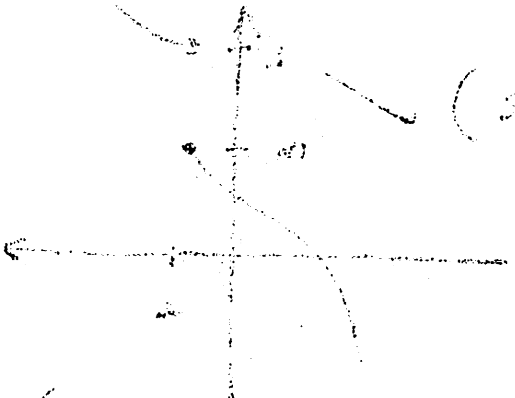
① (a, Q) ✓

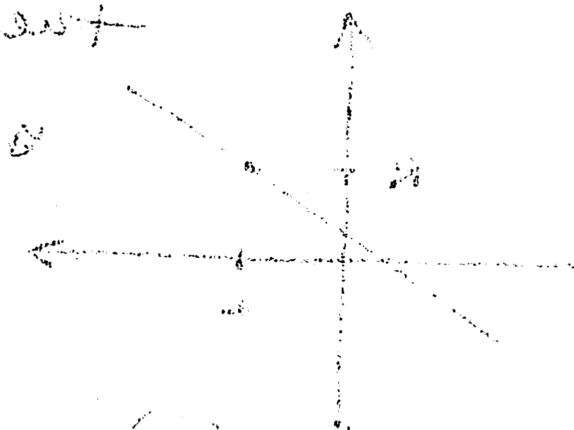
② $\lim_{x \to a} f(x) = m$ ✓

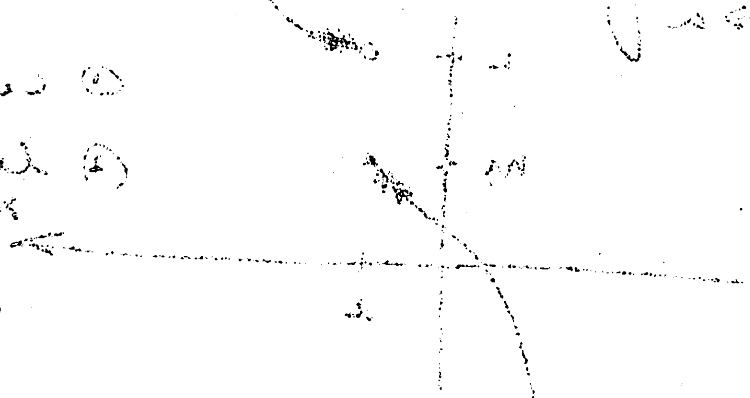
③ $\lim_{x \to a} f(x) \stackrel{?}{=} f(a)$

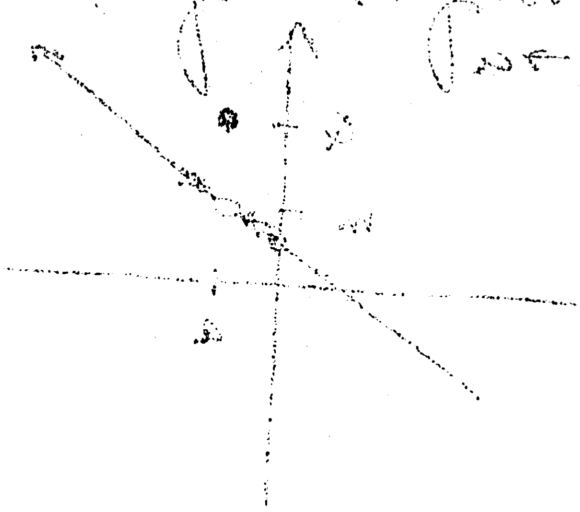
$m \neq Q$

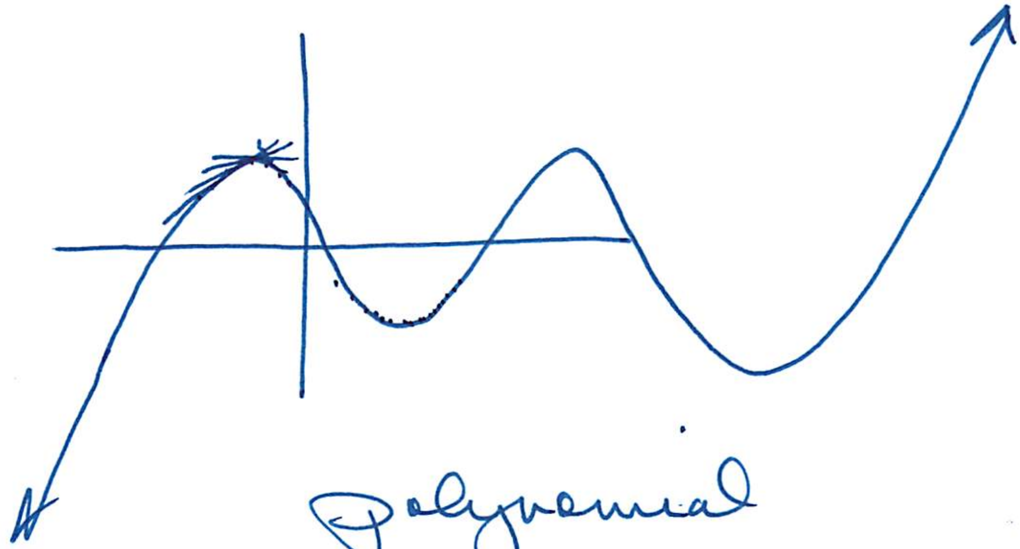
∴ DISCON







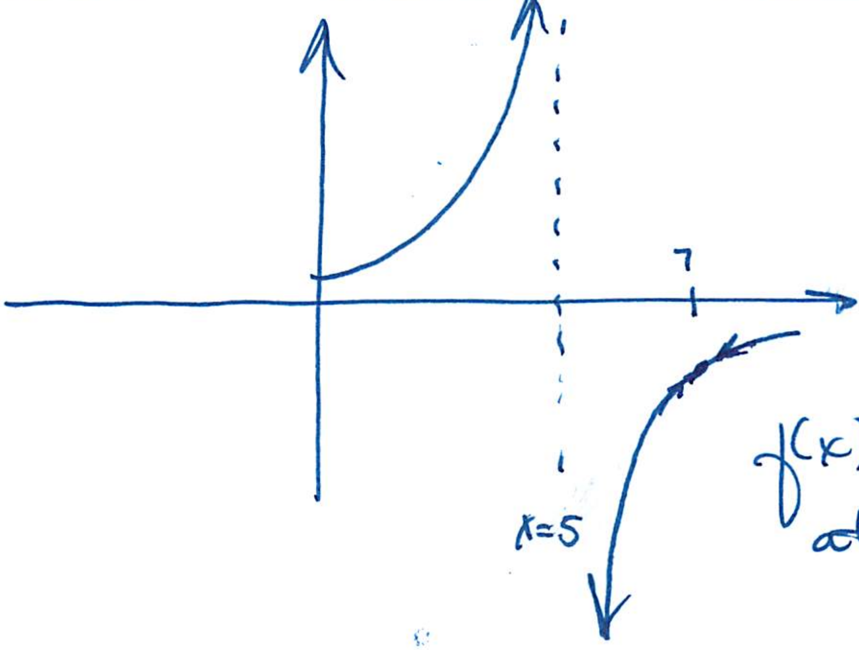




Polynomial

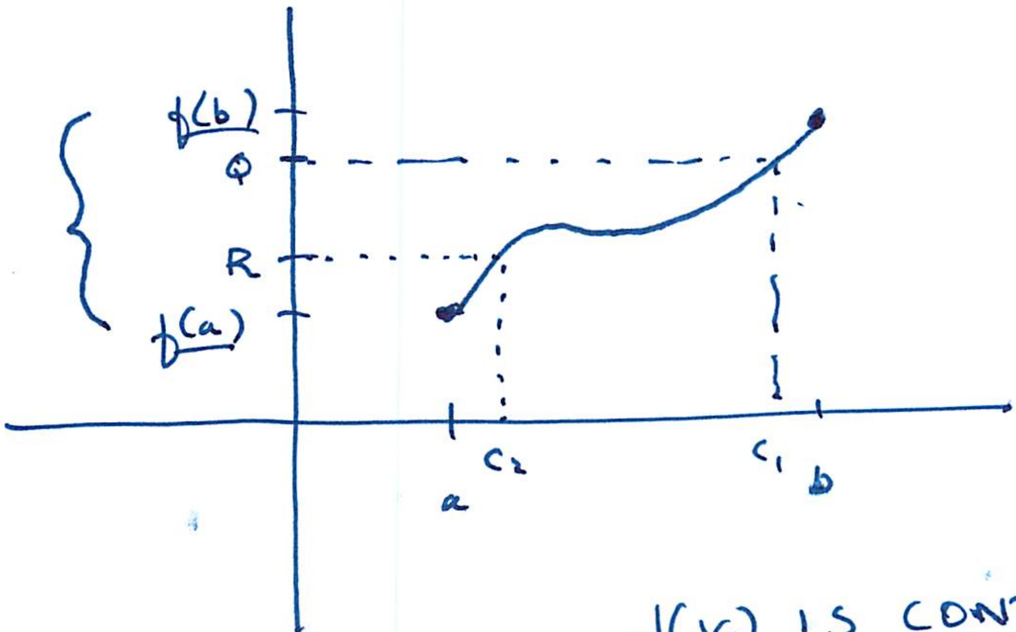
1 CONTIN.

2 "SMOOTH"



f(x) continuous at x = 7

INTERMEDIATE VALUE THEOREM (I.V.T.)



$f(c_1) = Q$

$f(c_2) = R$

$f(x)$ IS CONTIN.