MA141 - 005

Wednesday, January 16

ch 0:

parametric eq:

$t \in [0, \pi/4]$

$$\left.\begin{aligned} x &= 4\cos t \\ y &= 3\sin t \end{aligned}\right\} \qquad \begin{aligned} \frac{x}{4} &= \cos t \\ \frac{y}{3} &= \sin t \end{aligned}$$

$$\longrightarrow (\sin t)^2 + (\cos t)^2 = 1$$

$$\left(\frac{y}{3}\right)^2 + \left(\frac{x}{4}\right)^2 = 1$$

$$\frac{y^2}{9} + \frac{x^2}{16} = 1$$

$x = \sec t$

$y = \tan t$

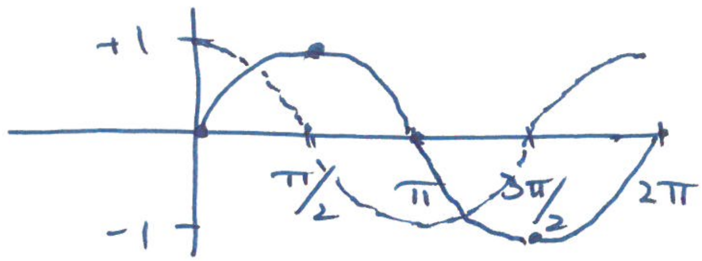
$$\frac{s^2}{c^2} + \frac{c^2}{c^2} = \frac{1}{c^2}$$

$$(\tan t)^2 + 1 = (\sec t)^2$$

$$(y)^2 + 1 = (x)^2$$

$$\frac{s^2}{s^2} + \frac{c^2}{s^2} = \frac{1}{s^2} \qquad 1 + \cot^2 t = \csc^2 t$$

sine & cosine:



$(x+h)^0$ 1

$(x+h)^1$ 1 1

$(x+h)^2$ 1 2 1

1 3 3 1

1 4 6 4

$$(x+h)^4$$

$$= x^4 + 4x^3 \cdot h + 6x^2 \cdot h^2 + 4xh^3 + h^4$$

$$\frac{3 \cdot 4}{2} \qquad \frac{2 \cdot 6}{3}$$

$$(x+h)^{21}$$

$$= x^{21} + 21 x^{20} \cdot h +$$

$$\frac{20 \cdot 21}{2}$$

$$f(x) = \frac{4x}{x-2} \qquad y = \frac{4x}{x-2}$$

find $f^{-1}(x)$:

↑ inverse of $f(x)$

$\underline{\text{INV}}$:

$$\frac{x}{1} = \frac{4y}{y-2}$$

① switch x & y

② solve for y

$$4y = x(y-2)$$

$$4y = xy - 2x$$

$$2x = xy - 4y$$

$$\frac{2x}{x-4} = \frac{y(x-4)}{x-4}$$

HA: $4 = y$
VA: $x = 2$

ORIG: $\boxed{f(x) = \frac{4x}{x-2}}$

$$\lim_{x \to \infty} \frac{2x}{x-4} = 2 \qquad y = \boxed{\frac{2x}{x-4} = f^{-1}(x)}$$

HA: $2 = y$
VA: $4 = x$

$$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$$

↓

$$f(f^{-1}(x)) = f\left(\frac{2x}{x-4}\right) = \frac{4\left(\frac{2x}{x-4}\right)}{\frac{2x}{x-4} - 2}$$

$$\frac{4\left(\frac{2x}{x-4}\right)}{\frac{2x}{x-4}-\frac{2(x-4)}{x-4}} = \frac{\frac{8x}{x-4}}{\frac{2x-2(x-4)}{x-4}}$$

$$= \frac{8x}{\cancel{x-4}} \cdot \frac{\cancel{x-4}}{2x-2(x-4)} = \frac{8x}{\cancel{2x}-\cancel{2x}+8} = x$$

$f: \{(3, 8) \ldots\}$

$f^{-1}: \{(8, 3), \ldots\}$

$(f \circ f^{-1})(8) = f(f^{-1}(8)) = f(3) = \underline{8}$

$(f^{-1} \circ f)(3) = f^{-1}(f(3)) = f^{-1}(8) = \underline{3}$

Ch 1: LIMITS

"closer and closer" ϵ & δ

$$\lim_{x \to a} \boxed{f(x)} = L$$

as x gets "closer and closer" to a

$x \to 3$ (2-sided limit)

(1) 2.9, 2.99, 2.999, ...

(2) 3.1, 3.01, 3.001, 3.0001, ...

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2}$$

x "approaches" 2 $(x \neq 2)$

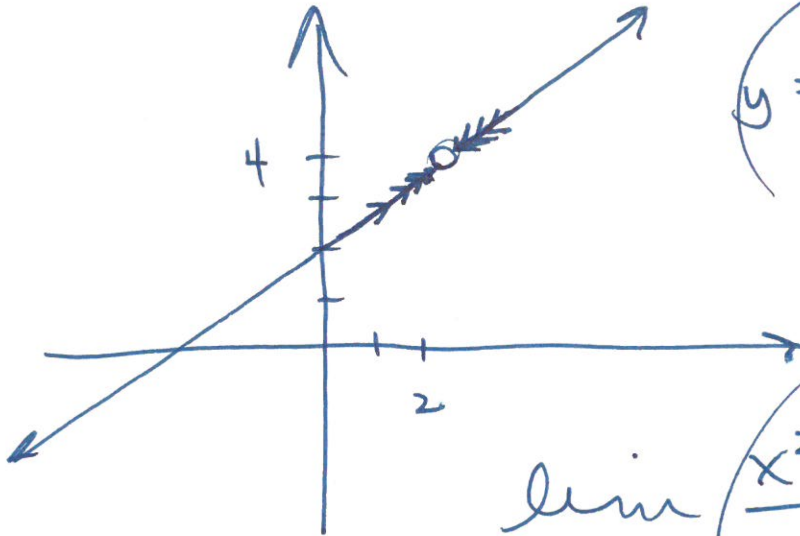
$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} = \lim_{x \to 2} (x+2) = \underline{4}$$

$(x \neq 2)$

$y = x+2$

$$\boxed{y = \frac{x^2-4}{x-2}} = x+2$$

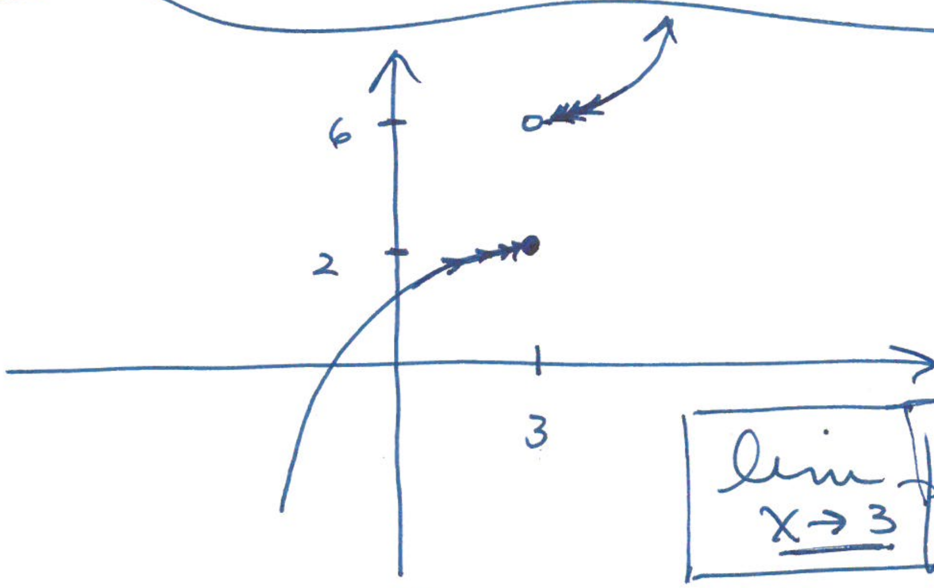
$(x \neq 2)$



$$y = \frac{x^2-4}{x-2} = x+2$$

$(x \neq 2)$

$$\lim_{x \to 2} \frac{x^2-4}{x+2} = 4$$



$$\boxed{\lim_{x \to 3} f(x) = \text{D.N.E.}}$$

NO LIMIT

$$\lim_{x \to 3^-} f(x) = \underline{2}$$

from the left

$$\lim_{x \to 3^+} f(x) = \underline{6}$$

from the right

$\underline{a}$ limit does not exist

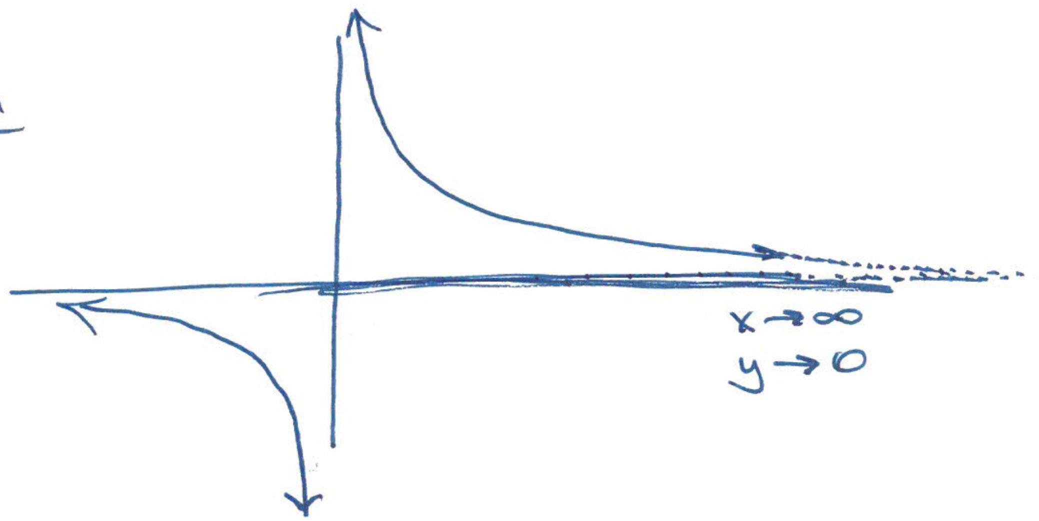
$$\lim_{x \to \infty} \boxed{\left(\frac{1}{x}\right)} = 0 \qquad \underline{y=0} \quad y = \frac{1}{x}$$

increase without bound

$$xy = 1$$

$$\frac{1}{10} \quad \cdots \quad \frac{1}{10{,}000} \quad \cdots\cdots \quad \frac{1}{1{,}000{,}000{,}000} \quad \cdots\cdots$$

lim



$f(x) = \dfrac{3x-7}{5x^2+2x-4}$

horizontal asymptote

$$\lim_{x \to \infty} \boxed{\frac{3x-7}{5x^2+2x-4}}$$

$$\lim_{x \to \infty} \frac{\dfrac{3x}{x^2} - \dfrac{7}{x^2}}{\dfrac{5x^2}{x^2} + \dfrac{2x}{x^2} - \dfrac{4}{x^2}}$$

$$\lim_{x \to \infty} \frac{\cancel{\dfrac{3}{x}}^{\to 0} - \cancel{\dfrac{7}{x^2}}^{\to 0}}{\underline{5} + \cancel{\dfrac{2}{x}}_{\to 0} - \cancel{\dfrac{4}{x^2}}_{\to 0}} = \frac{0}{5} = 0$$

$$\lim_{x\to\infty} \frac{2x-3}{\boxed{5x+8}}$$

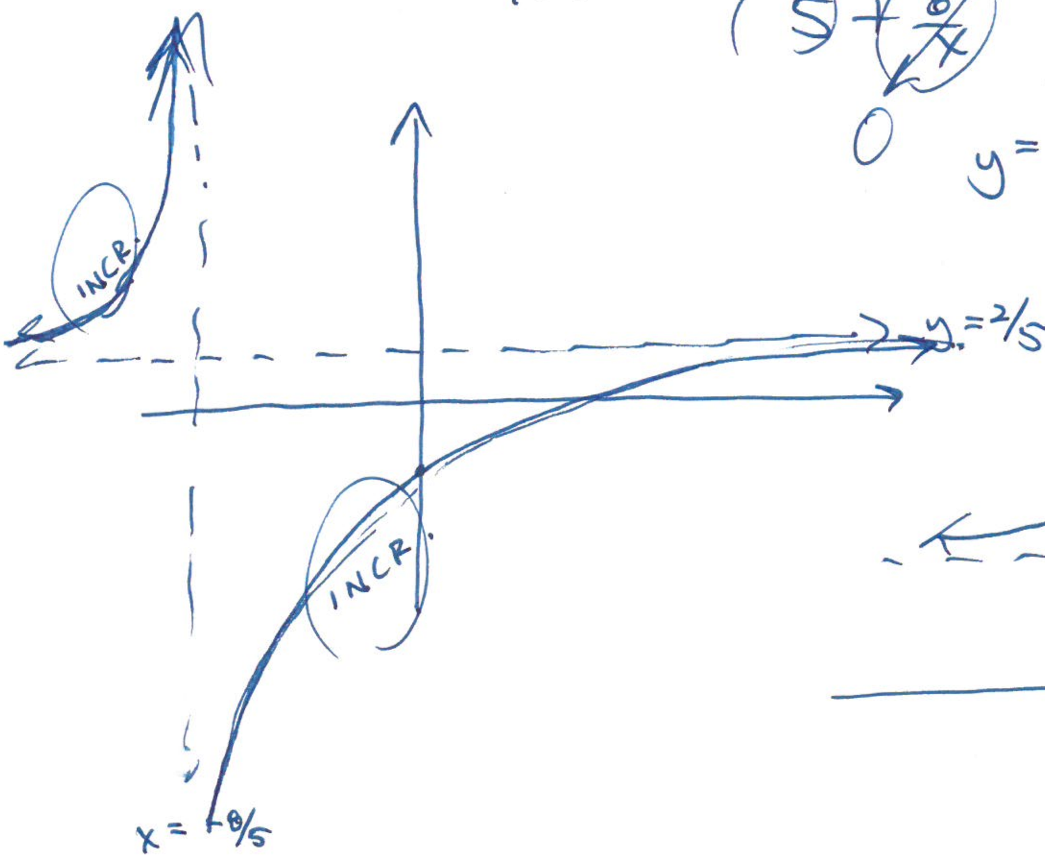
$$f(x) = \frac{2x-3}{5x+8}$$

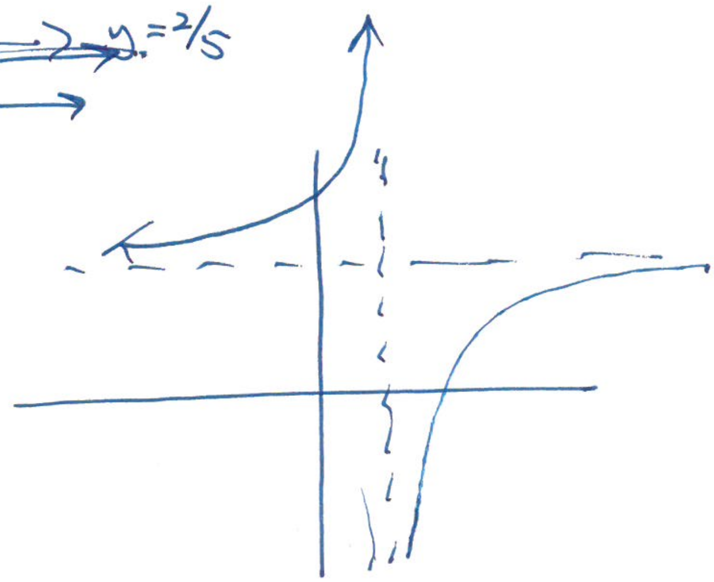
$$\lim_{x\to\infty} \frac{\frac{2x}{x} - \frac{3}{x}}{\frac{5x}{x} + \frac{8}{x}}$$

$x = \frac{-8}{5}$ is V.A.

$$\lim_{x\to\infty} \frac{2 - \cancel{\frac{3}{x}}^{\to 0}}{5 + \cancel{\frac{8}{x}}_{\to 0}} = \frac{2}{5}$$

$y = 2/5$ is H.A.





formal DEF. OF LIMIT.

if $|x - x_0| < \delta$

then $|f(x) - L| < \epsilon$